KARLSTADS UNIVERSITET

KANAZAWA UNIVERSITY

3rd Karlstad-Kanazawa Workshop
in applied Analysis
2025

Organizers: Nicklas Jävergård, Adrian Muntean, Guilia Rui

Date: 2 December, 2025

Place: Karlstad University, Room 21E205

Schedule

Time	**Event**
08:45–09:00	Opening
09:00–09:40	Giulia Rui, *Sulphation across scales: stochastic particle models, propagation of chaos, and effective PDE limits*
09:40–10:10	Masato Kimura, *Overdamped and elastic particle dynamics in bounded domains*
10:10–10:40	Fika!
10:40–11:20	Hirofumi Notsu, *Virtual physical reservoir computing with mathematical models*
11:20–12:00	Abhijit Guchhait, *Convergent Micro-Macro Iterative Scheme For Linear Elliptic Two-Scale System With Missing Boundary Data*
12:00–13:00	Lunch
13:00-15:00	Activity!
15:00–15:40	Martin Lind, *On equidistribution and related topics*
15:40–16:20	Nicklas Jävergård, *3D morphology formation of a ternary mixture during evaporation, well-posedness of a numerical scheme*
16:20–16:30	Closing remarks

Sulphation across scales: stochastic particle models, propagation of chaos, and effective PDE limits

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Air-pollution–induced degradation of calcium-carbonate substrates is primarily driven by sulphation, whereby sulfuric acid diffuses into the material and triggers a reaction converting calcium carbonate into gypsum. To analyze the multiscale structure of this process, we formulate a hybrid stochastic–continuum model. At the microscopic level, acid molecules are represented by an interacting particle system governed by stochastic differential equations with strongly singular Lennard–Jones drift, nonlocal environmental forcing, and Poissonian reaction jumps that model particle consumption. The calcium-carbonate and gypsum fields evolve through random ODEs coupled to the empirical particle measure.

We establish global well-posedness for this singular stochastic system via a probabilistic regularization procedure. Under suitable structural simplifications, we identify the mean-field behavior: an entropy-based approach yields a quantitative propagation-of-chaos result for the particle–reaction–environment system. The limit equation is a nonlinear reaction–advection–diffusion PDE featuring a nonlocal drift. Our current work focuses on homogenizing the resulting limiting dynamics in heterogeneous media to obtain effective macroscopic corrosion laws.

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Overdamped and elastic particle dynamics in bounded domains

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In this talk, we present analytical frameworks for two types of particle dynamics confined to bounded domains: overdamped motion and elastic collisions at the boundary. The first part concerns an overdamped system modeled by a first-order ODE, where particles are prevented from leaving the domain through instantaneous changes in velocity at the boundary. The second part focuses on a second-order ODE system in which interactions with the boundary are described by perfectly elastic reflections. Although these boundary rules are widely used in particle methods, rigorous notions of solutions for such constrained dynamics have been limited. Our results establish existence and (in part) uniqueness of solutions in possibly nonconvex domains, thereby providing a solid mathematical foundation for boundary conditions in particle-based numerical methods. This talk is based on joint work with Zhenxing Yang and Patrick van Meurs (Kanazawa University).

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Virtual physical reservoir computing with mathematical models

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Physical systems can be reinterpreted as powerful computational resources through the framework of reservoir computing [1, 2, 3], an information-processing paradigm that exploits the input-driven transient dynamics of high-dimensional dynamical systems. Focusing on the intrinsic information-processing capabilities of such systems, we introduce two "virtual" realizations of physical reservoir computing. The first system is the flow past a circular cylinder governed by the two-dimensional incompressible Navier–Stokes equations [4], and the second system is a forced Kuramoto reservoir [5] governed by the Kuramoto model [6], a ubiquitous model for phase locking, specifically the version that includes a forcing term to enable synchronization to an external input [7, 8]. Further details will be provided in the talk. The contents of the talk are based on [4, 5].

Acknowledgments: This work is supported by JST CREST Grant Number JPMJCR2014.

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Convergent Micro-Macro Iterative Scheme For Linear Elliptic Two-Scale System With Missing Boundary Data

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We study a coupled linear elliptic system posed on a two-scale microscopic and macroscopic domain, which originates for description of nutrients transport in plants. The model captures the interaction between microscopic and macroscopic unknowns, coupled through a linear transmission condition along the boundary of the microscopic interfaces.

The macroscopic data are only partially known. To recover missing boundary data, we introduce an alternating Dirichlet-Neumann scheme on selected boundary segments. We prove that our iterative scheme is well-posed in a weak sense. Our main result is a quantitative energy estimate that ensures the convergence of the iterative scheme to the unique weak solution.

Acknowledgments: This is financially supported by the Swedish Research Council with grant nr. 2024-05606.

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On equidistribution and related topics

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For the last few years, I have been thinking of and on about questions related to the concept "equidistribution". In the simplest setting, a numerical sequence $\{\xi_n\}_{n=1}^{\infty} \subset [0,1]$ is called equidistributed in $[0,1]$ if

$$\lim_{N\to\infty} \frac{\text{cardinality of } \{\xi_n : 1 \le n \le N\} \cap J}{N} = \text{length}(J)$$

for every interval $J \subseteq [0,1]$. I will discuss some of my results related to

(1) asymptotics of the discrepancy of certain equidistributed sequences defined in arithmetical terms involving primes or semiprimes [1, 2],
(2) approximation errors for the QMC method of numerical integration [3].

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3D morphology formation of a ternary mixture during evaporation, well-posedness of a numerical scheme

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The importance of renewable energy cannot be overstated. Organic solar cells is one cheap and scalable technology that can play a role in providing us with green energy. The efficiency and stability of organic solar cells depend on the morphology of the acceptor and donor phase in the photo active layer. The morphologies emerge as the polymeric phases seperate from each other in the presence of a solvent. In this talk we present our studies of a coupled system of partial differential equations that originates as one particular hydrodynamic limit of the Blume-Capel model with Kawasaki dynamics. This model is well-posed. We show the well-posedness of a semi-discrete finite volume scheme to approximate the solution. A fully discrete version of the scheme is used to obtain simulation results with and without evaporating the solvent in 3 spatial dimensions. Empirically estimated convergence rates and stability with respect to the inverse temperature parameter complement the discussion.

Acknowledgments: NJ and AM are involved in Swedish Energy Agency's project Solar Electricity Research Centre (SOLVE) with grant number 52693-1. ENMC thanks the PRIN 2022 project "Mathematical Modelling of Heterogeneous Systems (MMHS)", financed by the European Union - Next Generation EU, CUP B53D23009360006, Project Code 2022MKB7MM, PNRR M4.C2.1.1. ENMC thanks, also, the Mathematics Department of the Karlstad University for warm hospitality and GNFM–INDAM.

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