FAKULTETEN FÖR HÄLSA, NATUR- OCH TEKNIKVETENSKAP
INSTITUTIONEN FÖR MATEMATIK OCH DATAVETENSKAP
MATEMATIK

Mottagare/Receiver

Datum/Date
18/10/2025

Crash course "*Asymptotic methods for hybrid dimension modeling of thin tube networks*"

Speaker: Grigory Panasenko (prof. dr., Institute of Applied Mathematics, Vilnius University, Lithuania and Institute Camille Jordan UMR CNRS 5208, University Jean Monnet, Saint-Etienne, France)

Intended audience: PhD students, researchers, MSc students

Content of the course: Partial differential equations in thin domains combining thin plates and thin rods or pipes (so-called multi-structures) are extensively studied in mathematical solid and fluid mechanics (see e.g. [1-3]). In particular, in [3-6] the so-called thin tube structures were introduced as geometrical models for networks of thin blood vessels, tubes in catalytic converters, pipelines etc. The goal of the mini-course is to introduce the mathematical tools for constructing reduced models for various processes in thin tube structures. Such reduced models allow to importantly accelerate the computations of such processes and they may be used in machine learning.

1. Motivation: mathematical modeling of blood circulation in the heart and blood vessel networks.
2. Main equations: heat (diffusion) equation, Stokes and Navier-Stokes equations.
3. Dimension reduction of the Neumann problem for the conductivity equation in thin domain.
4. Asymptotic partial decomposition of thin domain for the Neumann problem.
5. Dimension reduction of the Dirichlet problem for the conductivity equation in thin domain.
6. Asymptotic partial decomposition of thin domain for the Dirichlet problem
7. Reduced models for the Navier-Stokes equations in thin tube structure and modeling of blood flow in networks of blood vessels (partially in the frame of the KAAS seminar).

References: The course is based on the following works:

1. P.G. Ciarlet, *Plates and Junctions in Elastic Multi-structures. An Asymptotic Analysis*, Masson, Paris, 1990
2. V. Kozlov, V. Maz'ya, A. Movchan, *Asymptotic Analysis of Fields in Multi-Structures*, Oxford Mathematical Monographs., Oxford Science Publications, The Clarendon Press, Oxford University Press, New York, 1999.
3. G. Panasenko, *Multi-scale Modeling for Structures and Composites*, Springer, Dordrecht, 2005.
4. G. Panasenko, *Introduction to Multiscale Mathematical Modeling*, World Scientific, New Jersey/London/Singapore/Beijing/Shanghai/Hong Kong/Taipei/Chennai/Tokio, 2022, 158 pp.

Course organization details and time schedule: The course is self-contained but it will run at an alert tempo. Homework exercises will also be assigned for those willing to deepen their knowledge in the proposed content. The course has 3 offline moments (each à 2 hours) and 1 hybrid moment (KAAS seminar, 45 minutes + questions). The schedule is as follows:

Monday, 27 of October, 2025, 10h-12h, Karlstad University, room 1D230

Tuesday, 28 of October, 2025, 10h- 12h, and 14h-16h, Karlstad University, room 21E415A

Wednesday, 29 of October, 2025, 10h30-11h15, KAAS seminar,
https://www.kau.se/matematik/kaas

Title and abstract of the KAAS seminar:

TALK-153
When: 29 Oct 2025, 10:30-11:15
What: Asymptotic and numerical analysis of non-Newtonian flows in networks of blood vessels
Who: Grigory Panasenko (Institute of Applied Mathematics, Vilnius University, Lithuania and Institute Camille Jordan UMR CNRS 5208, University Jean Monnet, Saint-Etienne, France)
Where: https://kau-se.zoom.us/j/61616693592

Partial differential equations in thin domains combining thin plates and thin rods or pipes (so-called multi-structures) are extensively studied in mathematical solid and fluid mechanics (see e.g. [1-3]). In particular, in [3-6] the so called thin tube structures were introduced as geometrical models for networks of thin blood vessels, tubes in catalytic converters, pipelines etc. The asymptotic analysis of the viscous Newtonian flows in these structures allowed to introduce the hybrid dimension models. They combine one-dimensional and multidimensional description of the flow with asymptotically exact coupling conditions between 3D and 1D parts of the model (see [4,5] and a recent monograph [6]). Another approach for junction of models of different dimensions for blood flow in arteries was proposed in [7, 8]. Thus, hybrid dimension models provide the one-dimensional description in the main part of the domain and make small full-dimensional zooms. These zooms give detailed description of the flow in the zones of interest such as the bifurcations of vessels, zones of blood clot formation, stents and so on. The hybrid dimension models allow substantially accelerate computations without loss of accuracy. In [3-6] the so called Newtonian rheology was considered : the Navier-Stokes equations with constant viscosity. However, it is well-known that the blood as well as melted polymers exhibit non-Newtonian rheology, when the viscosity depends on the gradient of velocity (shear rate). In particular, the numerical simulations for the blood show the difference about 10-15% between the results obtained via Newtonian and non-Newtonian models of flows. The talk will present recent results on asymptotic analysis and partial asymptotic reduction for the equations of non-Newtonian flows in thin tube structures, as well as some numerical experiments for real-life

networks of vessels. These results are obtained in collaboration with K.Pileckas (see [9]). They are justified via asymptotic analysis of the full-dimensional problem in the whole domain of the flow and the proof of estimates for the difference between the exact solution of the full-dimensional problem and the solution to the hybrid dimension model.

References for the KAAS seminar:

1. P.G. Ciarlet, *Plates and Junctions in Elastic Multi-structures.* An Asymptotic Analysis, Masson, Paris, 1990.
2. V. Kozlov, V. Maz'ya, A. Movchan, *Asymptotic Analysis of Fields in Multi-Structures*, Oxford Mathematical Monographs., Oxford Science Publications, The Clarendon Press, Oxford University Press, New York, 1999.
3. G. Panasenko, *Multi-scale Modeling for Structures and Composites*, Springer, Dordrecht, 2005.
4. G. Panasenko, *Asymptotic expansion of the solution of Navier-Stokes equation in a tube structure*, C.R. Acad. Sci. Paris, 326, IIb, 1998, 867—872.
5. G. Panasenko, *Partial asymptotic decomposition of domain: Navier-Stokes equation in tube structure*, C.R. Acad. Sci. Paris, 326, IIb, 1998, 893—898.
6. G. Panasenko, K. Pileckas, *Multiscale Analysis of Viscous Flows in Thin Tube Structures*, Birkhauser, Springer Nature Switzerland AG, 2024.
7. L. Formaggia, A. Veneziani, *Reduced and Multiscale Models for the Human Cardiovascular System*. Lecture Notes, VKI, Brussels, 2003.
8. L. Formaggia, A. Moura, F. Nobile, *On the stability of the coupling of 3D and 1D fluid-structure interaction models for blood flow simulations*, M2AN, 2007, 743-769.
9. G. Panasenko, K. Pileckas, *Partial asymptotic dimension reduction for steady state non-Newtonian flow with strain rate dependent viscosity in thin tube structure,* J.Math. Fluid. Mech., 25:11, 2023.

Recognition of active participation: A participation certificate will be issued upon request. PhD students who wish to obtain 2 ECTS for their participation in this course should indicate their intention at the beginning of the course. They are expected to actively engage during the course sessions, the KAAS seminar, and in completing the assigned homework exercises.

Contact person for organization matters: Participation in this course is free of charge. To facilitate organizational planning, please inform us in advance if you intend to attend.

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