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A new hybrid Boltzmann-BGK model: consistency, hydrodynamic limits and applications.

Abstract

The evolution of a gas is classically described by the Boltzmann equation and the con- tribution of interactions is modeled by proper integral nonlinear operators. Unfortunately this approach requires a high computational cost in simulations for gas mixtures due to a larger number of collision operators (one for each type of interaction) [1].

For such reason, alternative formulations have been proposed since the pioneering model for a single gas proposed by Bhatnagar, Gross and Krook [2], whose idea was to replace the integral nonlinear operator by a simpler linear one reproducing the relaxation of the system towards a Maxwellian state and recovering the usual conservations properties. The extension of this approach to mixtures is not unique; a consistent BGK model has been pro- posed recently in [3], where the sum of Boltzmann terms is replaced by a sum of relaxation operators, one for each couple of components.

In this talk we want to propose a new mixed model combining the positive features of Boltzmann and BGK descriptions, preserving the accuracy of Boltzmann description for a part of the collisional phenomenon and using the BGK approach for the remaining processes. The consistency of the model is proved, focusing in particular on the preservation of global momentum and total energy, convergence to a global (Maxwellian) equilibrium; moreover, the existence of a Lyapunov functional miming the entropy dissipation is guaranteed.

This model is very useful to describe different hydrodynamic regimes, like the one dom- inated by intraspecies collisions, typical for mixture whose components have very disparate masses (e.g. ions and electrons) [4]. For such regime, we derive macroscopic equations of Euler and Navier-Stokes type and we test them on the classical shock wave problem.

This work is in collaboration with M. Bisi, M. Groppi, E. Lucchin and A. Macaluso (University of Parma).

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