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2nd Karlstad-Kanazawa Workshop

in Applied Analysis

Organizers: Adrian Muntean, Michael Eden

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Date: March 15, 2023 Place: Karlstad University, Room 21E205

Schedule

8:45	Opening
9:00	Hirofumi Notsu
9:30	Rainey Lyons
10:00	Michal Beneš
10:30	*Fika and coffee*
11:00	Michael Eden
11:30	Yuki Karasawa
11:45	Surendra Nepal
12:00	*Lunch break and walk*
14:30	Grigor Nika
15:00	Masato Kimura
15:30	*Coffee break*
15:45	Vishnu Raveendran
16:00	Nicklas Jävergård

A total design of finite element schemes for flow problems

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In the development of numerical schemes for flow problems, especially in the framework of the finite element method, there are some key issues, e.g., the approximation of the convection term, the choice of a finite element, and the mesh control, where the last one is not special for flow problems. In this talk, we introduce our total design of finite element schemes for flow problems and related results, cf. [1–3].

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Modeling size structured coagulation-fragmentation equations in the space of Radon measures

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In this talk, we will cover a brief introduction to the tools and techniques used in modeling size structured population models in the space of Radon measures. We will cover the notion of weak solutions, possible norms, and try to motivate the use of measures in some applications. We will then introduce a size structured coagulation-fragmentation model which has been used to model the population dynamics of oceanic phytoplankton. We will then reformulate the model in the space of Radon measures, show the model is well-posed, and present some properties of solutions to this model. Along the way, we show that this model unifies the study of discrete and continuous coagulation-fragmentation equations. Finally, we will discuss numerical simulations of the model via the use of flux limiter finite difference schemes.

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Mesoscale model of binary blended cement paste/aggregate fresh concrete: analysis and homogenization result

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In this talk we discuss some theoretical aspects of a parabolic problem with an integral condition which is of interest when solving the so-called problem of hydratational heat in concrete massives. The problem of hydration and heat transport in fresh concrete is strongly coupled and non-linear, and therefore, difficult from theoretical as well as computational point of view. Concrete is assumed as a composite material with two periodically distributed mesoscale components, cement paste (Ω_c) and aggregates (Ω_a). We assume different thermal characteristics in Ω_c and Ω_a. Physically accurate results can be obtained using fine-scale simulations, which are, however, extremely time consuming. Therefore, there is an interest in developing new physically accurate and computationally effective models.

To capture the heterogeneous structure of concrete, the problem is formulated as an initial-boundary-value problem for the system of the parabolic equation and integral condition with highly oscillating transport coefficients. Existence of global weak solutions of the mesoscale problem is proved by means of semidiscretization in time deriving a priori estimates for discrete approximations needed for proofs of existence and convergence theorems. The uniqueness result is proven under some assumption on the regularity of the solution (which is also shown). Finally, a homogenized model is derived by an upscaling method from the mesoscale model. The coefficients for the homogenized model are obtained from the solution of a periodic cell problem.

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Effects of cooling fluids in tool grinding

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Tool grinding is a complex production process where grinding wheels are used to shape workpieces; during this process, cooling lubricants are applied with high pressures and in large quantities. Modeling the complex and highly dynamic interactions between grinding wheel and lubricants leads to the question of heat transfer at interfaces between porous media and surrounding fluid layers.

In this talk, we are investigating effective heat transfer conditions in the context of mathematical homogenization. We present results for two different cases:

(a) The solid part of the porous media is assumed to consist of disconnected inclusions. Here, we arrive at a one-temperature problem exhibiting a memory term.

(b) The solid matrix is assumed to be connected. Here, the limit problem is given by two-phase mixture model.

We compare and discuss these two limit models with several simulation studies both with and without convection.

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A finite element scheme for the Oldroyd-B model with a second-order approximation of the upper-convected time derivative

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An essential term in viscoelastic fluid models is the upper-convected time derivative (UCTD):

$$\overset{\triangledown}{\tau} := \frac{\partial \tau}{\partial t} + (u \cdot \nabla)\tau - (\nabla u)\tau - \tau(\nabla u)^{\top}. \tag{1}$$

Recently, a second-order approximation of UCTD in time has been proposed and implemented in a finite difference framework [1]. In this talk, we extend this idea and propose a new finite element scheme for treating UCTD in the context of the generalized Lie derivative. Two-dimensional numerical results are presented to see the experimental order of convergence for the scheme.

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Estimating the ingress of diffusants into rubbers with a random walk method for moving boundaries

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We are interested in the modeling, analysis, and numerical simulation of diffusants penetrating dense rubbers. For certain materials science scenarios arising in rubber technology, one-dimensional moving boundary problems with kinetic boundary conditions are capable of unveiling the large-time behavior of the diffusants penetration front, giving a direct estimate on the service life of the material. In this framework, we deal with the construction of a random walk approximation method to approximate numerically the concentration profile and the location of the sharp front. Essentially, our scheme decouples the targeted evolution system by solving via an explicit Euler method the ordinary differential equation corresponding to the evaluation of the speed of the moving boundary and then, as the next step, one deals with a diffusion problem by a random walk method. We present suitable random walk approximations and compare them to a finite element approach [1] with a controlled convergence rate [2]. Our numerical experiments recover well penetration depth measurements of an experimental setup.

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Influence of Internal Length Gradient (ILG) micromechanics on homogenized constitutive laws

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Our scientific inquiry will center on the development of homogenized models from periodically heterogeneous Cosserat media, taking into consideration the significant impact of microstructural size effects and complex geometric phenomena such as chirality. In particular, chirality is often associated with intricate gyroid morphology and is characterized by mirror symmetry. The classification of the homogenized equations relies upon the intricate hierarchy of four characteristic lengths that include the size of the heterogeneities, the Cosserat intrinsic lengths of the constituents, and the overall characteristic length of the domain. By delving into the multifaceted interplay between these intrinsic length scales, we can derive either an effective Cauchy continuum or an effective Cosserat continuum, which may or may not exhibit centrosymmetry.

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A phase field method for the problems of fluid-driven crack propagation in porous medium: hydraulic fracturing and desiccation cracking

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Fluid-driven crack propagation is an important issue related to the fracking well stimulation technique in these decades. There are two major issues related to fluid-driven crack propagation, namely: hydraulic fracturing and desiccation cracking. Hydraulic fracturing occurs due to the high water injection in a reservoir which causes open cracks along the fluid flow. Opposite to the hydraulic fracturing, desiccation cracking is reproduced by water evaporation along the material surface which can induce volume shrinkage in the material. We propose a phase field model for crack propagation due to fluid pressure which can be applicable for both hydraulic fracturing and desiccation cracking. The proposed model involves irrevesibility of the crack propagation, non-penetration condition of the crack surface, fluid pressure in the porous medium, and a natural energy dissipation property. This is a joint work with Sayahdin Alfat, Kanazawa University.

Homogenization of convection dominating non-linear Reaction-Diffusion-Convection problem in unbounded porous media

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We study the periodic homogenization of a reaction-diffusion problem with nonlinear drift posed in an unbounded perforated domain. We are interested in deriving rigorously the upscaled model equations and the corresponding effective coefficients for the case when the microscopic dynamics are linked to a particular choice of characteristic length and time scales that lead to a fast exploding nonlinear drift. The main mathematical difficulty lies in proving the two-scale compactness and strong convergence results needed for the passage to the homogenization limit. For, this we use two-scale compactness with drift, which is similar to classical two scale compactness result but defined in a moving coordinate. Later stage we expect to derive corrector type estimate for our problem. This is work in progress. Our setting is closely related to the works by Marusik-Paloka and Piatnitskii [2] as well as by Raveendran et al. [1].

Acknowledgments: The work of V.R. and A.M. is partially supported by the Swedish Research Council's project "*Homogenization and dimension reduction of thin heterogeneous layers*" (grant nr. VR 2018-03648).

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Remarks on a forward-backward nonlinear ODE system

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We propose a suitable iteration scheme to tackle a toy forward-backward ODE system, with potential application to economic systems. We prove its convergence and illustrate numerically the behavior of the solution.

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