Karlstad Applied Analysis Seminar (2022)

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Strong advection problems in turbulent diffusion

Abstract

In this talk, we shall attempt to give an overview of a new weak convergence type tool developed by myself, Thomas Holding and Jeffrey Rauch to handle multiple scales in advection-diffusion type models used in the turbulent diffusion theories. The equation that we consider models the evolution of a concentration field which is being advected by a prescribed incompressible fluid field and in the presence of molecular diffusion. Our mathematical setting is to consider advective fields of greater magnitude in comparison with the molecular diffusion. Our strategy, loosely speaking, is to recast the advection-diffusion equation in moving coordinates dictated by the flow associated with a mean advective field. Crucial to our analysis is the introduction of a fast time variable. We introduce a notion of convergence along mean flows which is a weak multiple scales type convergence – in the spirit of two-scale convergence theory used in homogenization. We have used ideas from the theory of homogenization structures developed by G. Nguetseng. Our main result gives a sufficient structural condition on the Jacobian matrix associated with the flow of the mean advective field which guarantees the homogenization of the original advection-diffusion problem. We also show the robustness of this structural condition by giving examples where the failure of such a structural assumption leads to a degenerate limit behaviour. More details on this new analytic tool in homogenzation theory and on the structural condition for the Jacobian matrix can be found in

T. Holding, H. Hutridurga, J. Rauch. Convergence along mean flows, SIAM J Math Anal., Volume 49, Issue 1, pp.222–271 (2017). In a sequel to the above work, we have addressed the growth in the Jacobian matrix – termed as Lagrangian stretching in Fluid dynamics literature – and its consequences on strong advection limit. To this effect, we introduce another kind of multiple scales convergence in weighted Lebesgue spaces. Our analysis demonstrates the presence of multiple time scales in certain turbulent diffusion models and the dynamics at each of these time scales helps us understand the underlying physical phenomena. The results in our work have some close links to Freidlin-Wentzell theory. Furthermore, this talk will illustrate the theoretical results via various interesting examples. We address some well-known advective fields such as the Taylor-Green cellular flows, the cat's eye flows and some special class (integrable) of the Arnold-Beltrami-Childress (ABC) flows. We will also comment on certain examples of hyperbolic or Anosov flows. The objective of this talk is to present both the above mentioned aspects of our work in an unified manner.