Design of 3D printable thin composite panels featuring an extension-bending coupling

Filippo AGNELLI

Department of Mechanical Engineering, McGill University, Montreal, QC, Canada

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Acknowledgement

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  - **Grégoire Allaire**, and **Charles Dapogny** CMAP, École polytechnique, France
  - **Chiara Daraio** and **Paolo Celli**, Caltech, USA

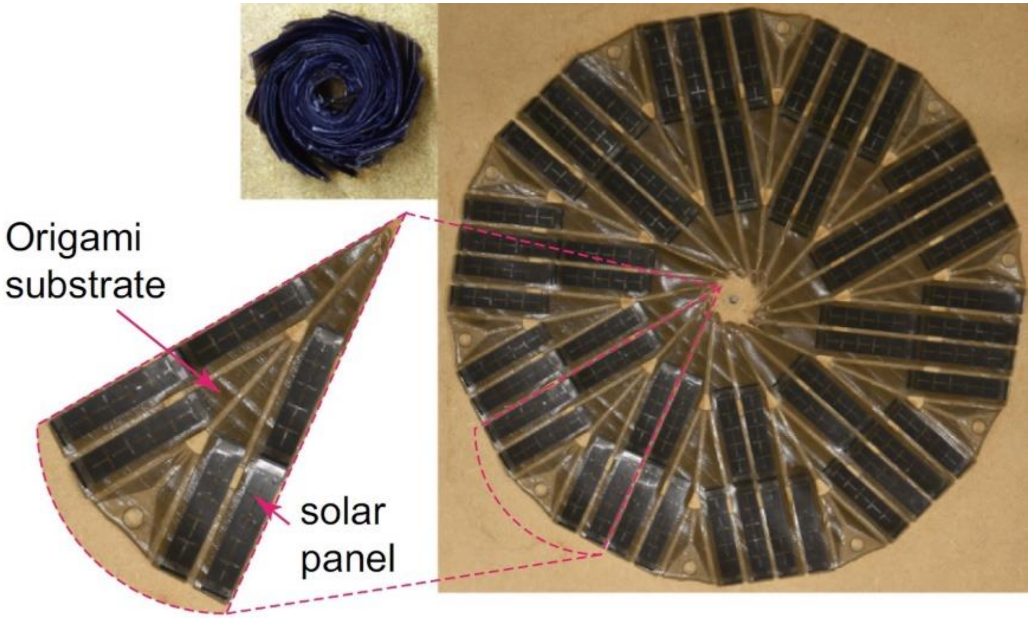
Shape-shifting panels in engineering application

- **Panel: Compound (cellular and/or laminate) & Slender (plate or shell) structure**
  - High strength-to-weight ratio
  - Simple manufacturing
  - Efficient storage and transportation

- **Shape-shifting structure**
  - Self-deployable (flat to 3d and converse)
  - Reconfigurable
  - Possible actuation / locomotion

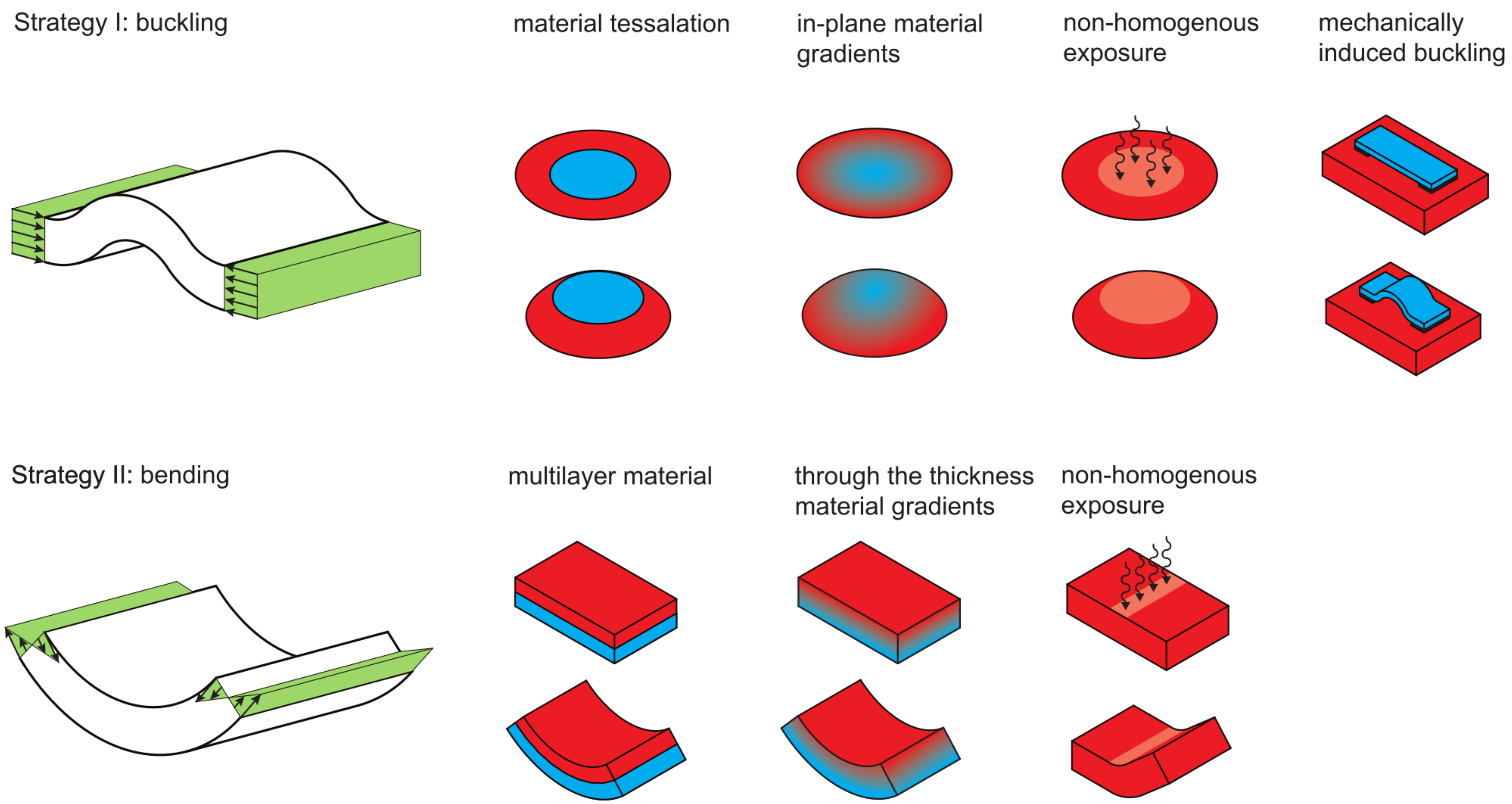
=> Shape-shifting panel
  - Wider design space
  - Multifunctionality

(Guseinov et al. *Nat. Commun.* 2020)



(Chen et al. *Phys. Rev. Appl.* 2019)

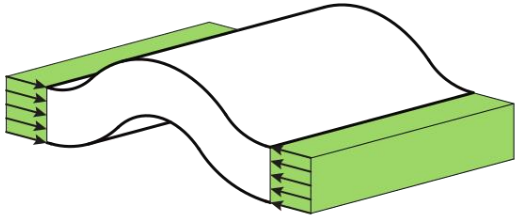
Shape-shifting structures: origins of the mechanism



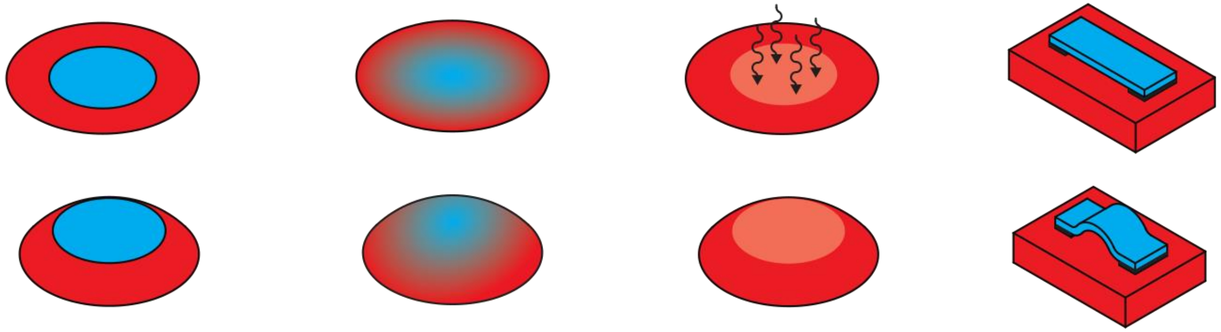
(van Manen, Janbaz & Zadpoor, *Mater. Today* 2018)

Shape-shifting structures: origins of the mechanism

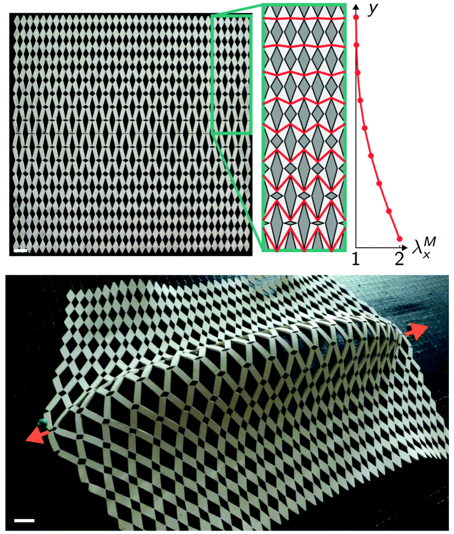
Strategy I: buckling



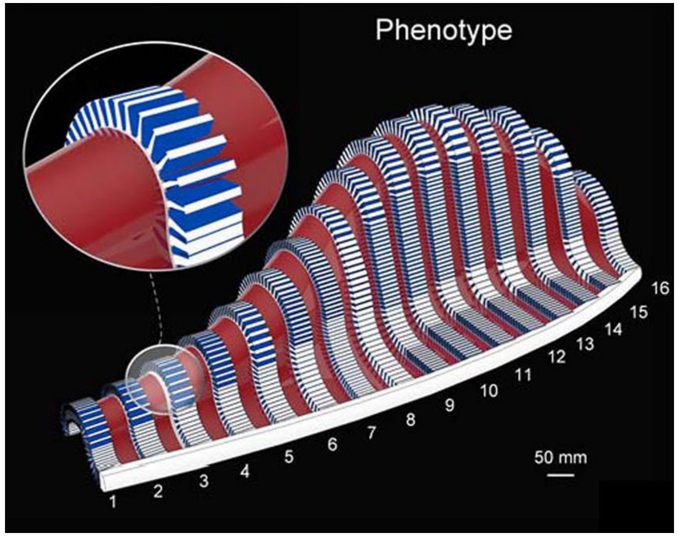
material tessalation | in-plane material gradients | non-homogenous exposure | mechanically induced buckling



(Siefert et al. *Soft Matter* 2020)



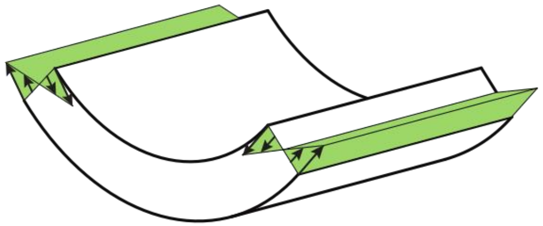
(Celli et al. *Soft Matter* 2018)



(Liu et al. *Sci. Rep.* 2019)

Shape-shifting structures: origins of the mechanism

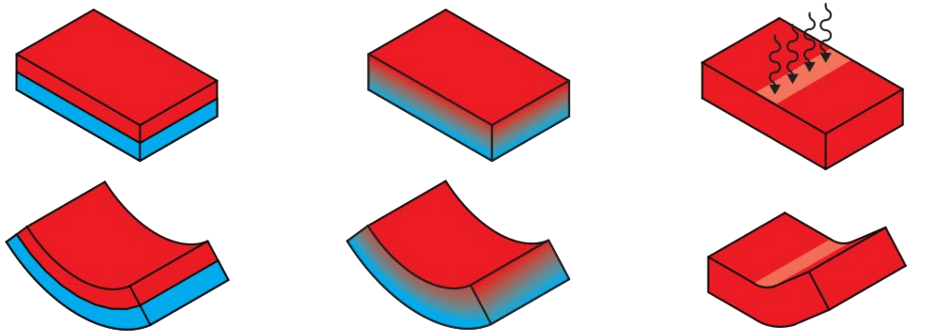
Strategy II: bending

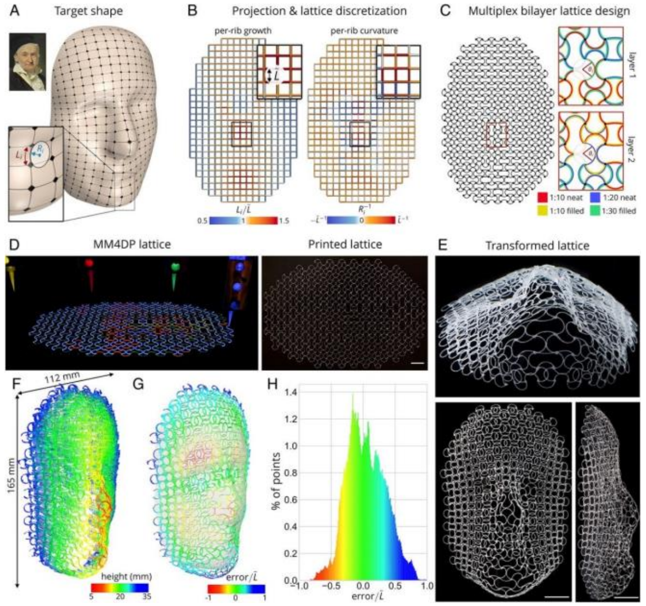


multilayer material

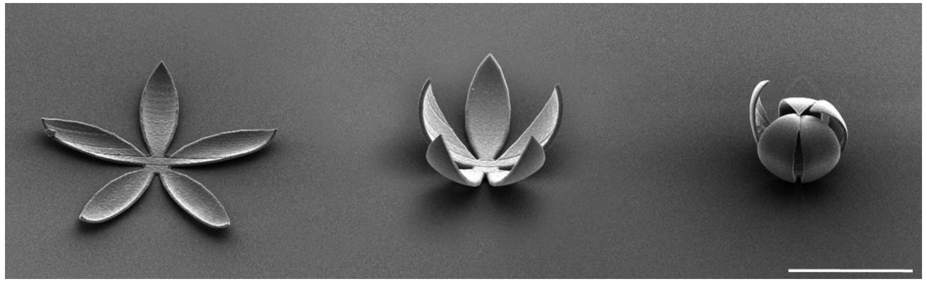
through the thickness material gradients

non-homogenous exposure

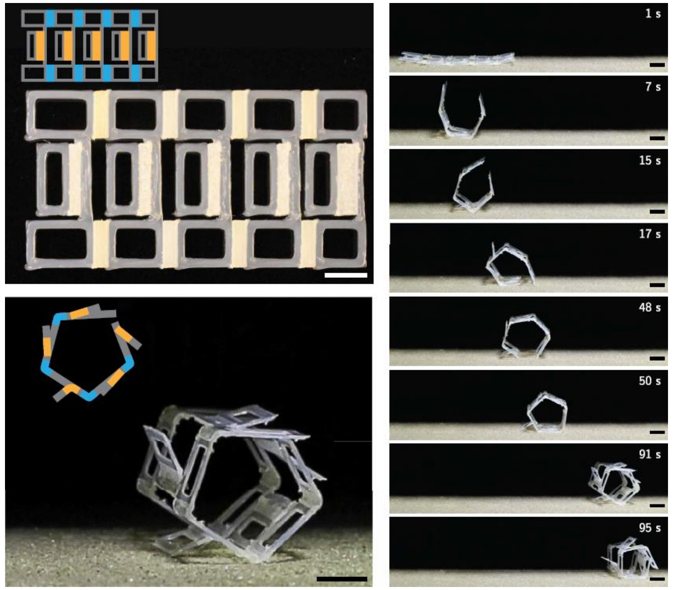




(Boley et al. *PNAS* 2020)



(Bauhofer et al. *Adv. Mater.* 2017)

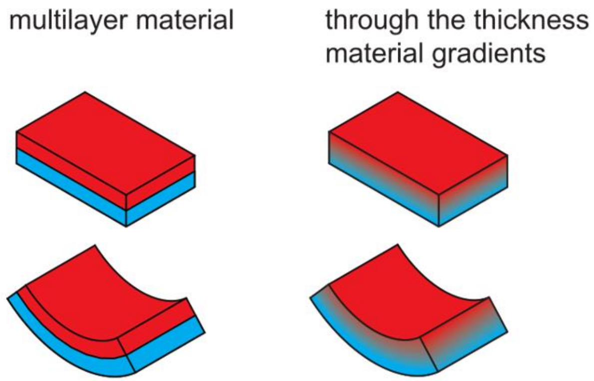


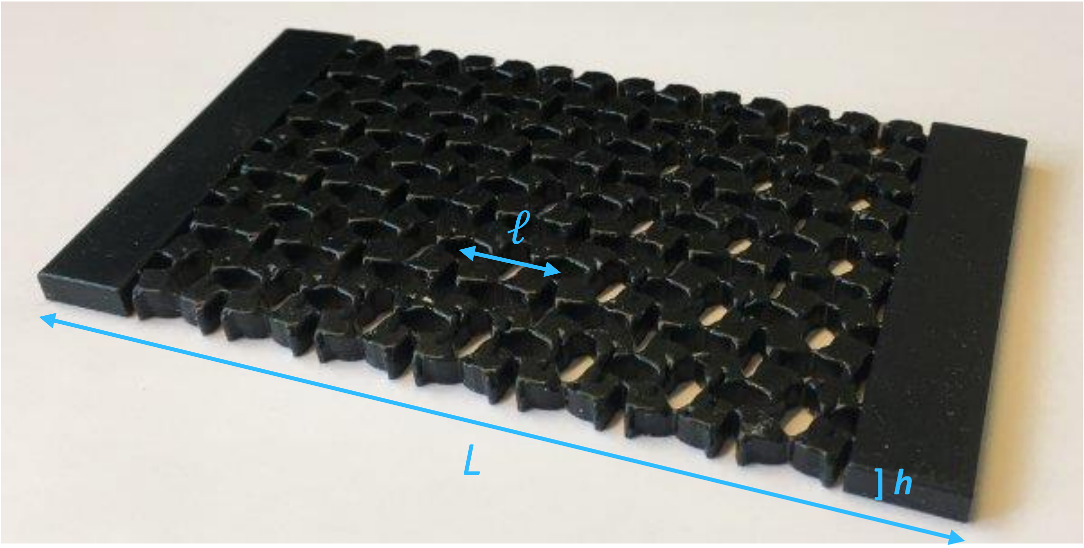
(Kotikian et al. *Sci. Robot.* 2019)

Design of 3D printable architectured sheets

Method:

- Impart a material gradient in the panel
  - Single material
  - Controlled material distribution





- Periodic structure

$$\epsilon = \frac{\ell}{L}$$

- Cellular aspect ratio

$$r = \frac{h}{\ell}$$

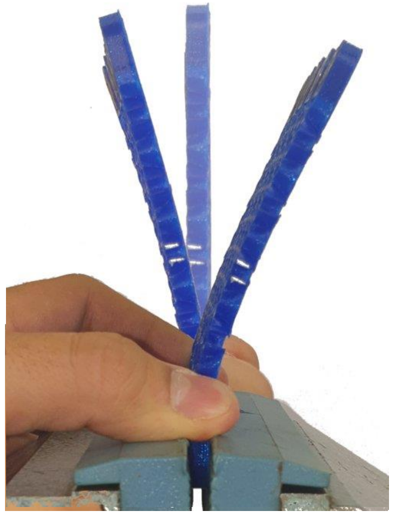
- $r \ll 1$: 2D plane stress state
- $r \approx 1$: 3D unit cell

Outline

- **Mechanical characterization of periodic panels**
  - Analysis of a ribbon-based periodic panel
    - Asymptotic homogenization theory
    - Generalized Kirchhoff-Love model

- **Topology optimization using a level set method**
  - Finding an elementary form associated to a target elastic stiffness
  - Shape derivative in the sense of Hadamard
  - Classical level set method

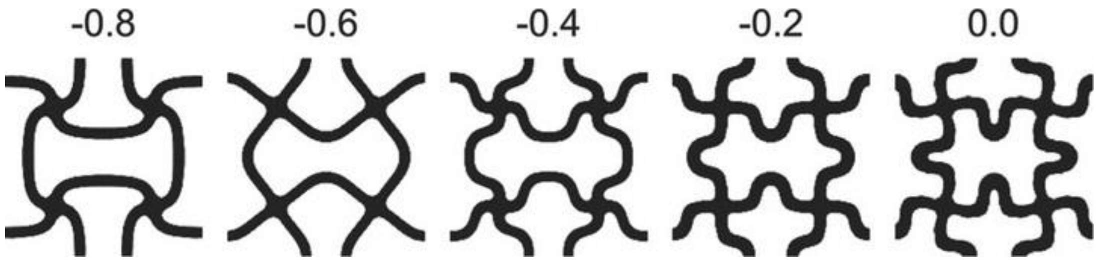
« Manual pinching »

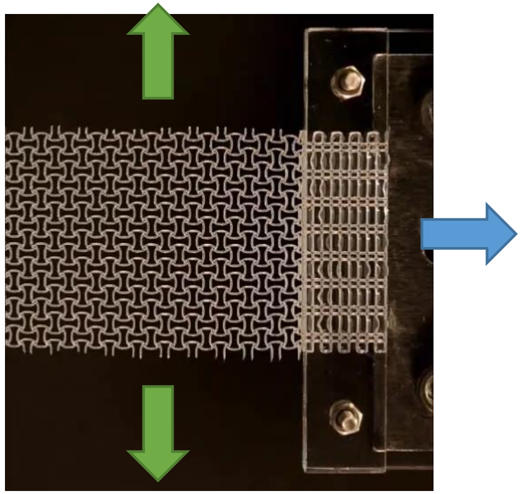


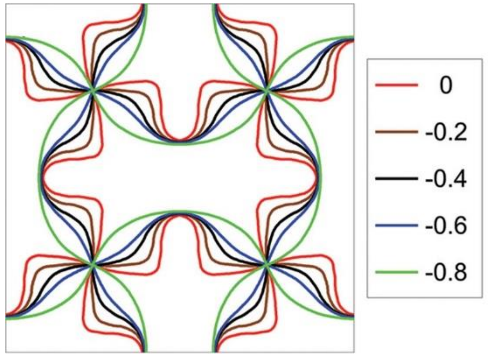
(A., Tricarico & Constantinescu *Extreme Mech. Lett.* 2021)

Families of auxetic lattices

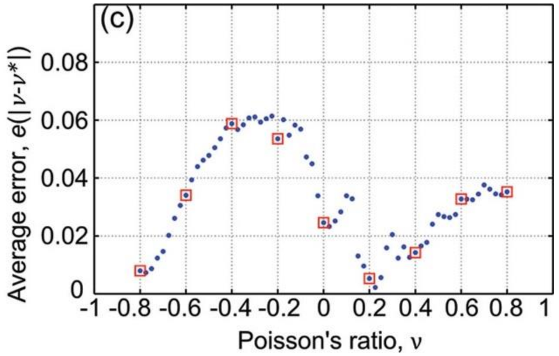
- Programmable Poisson's ratio over large deformations up to 20%



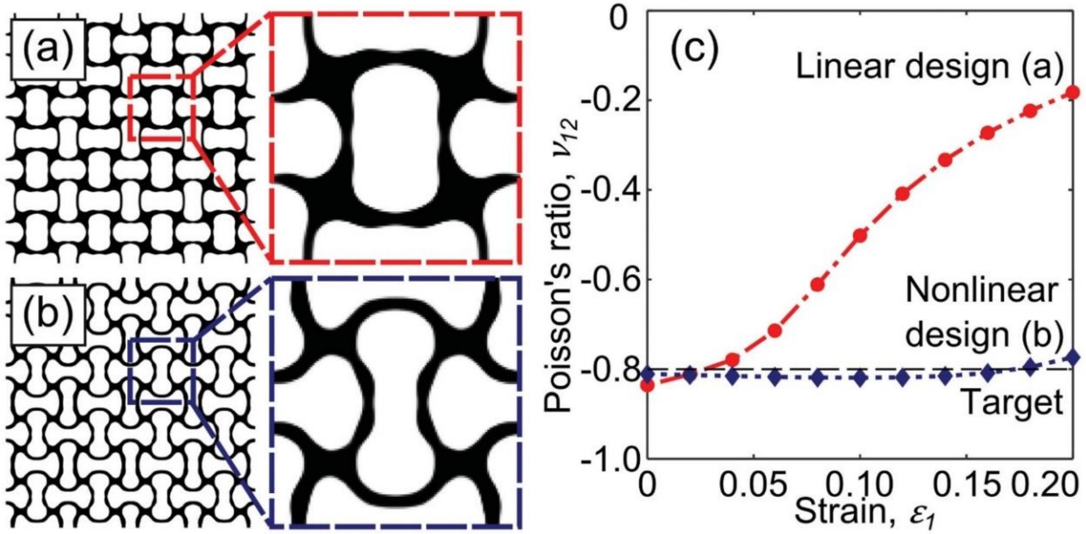




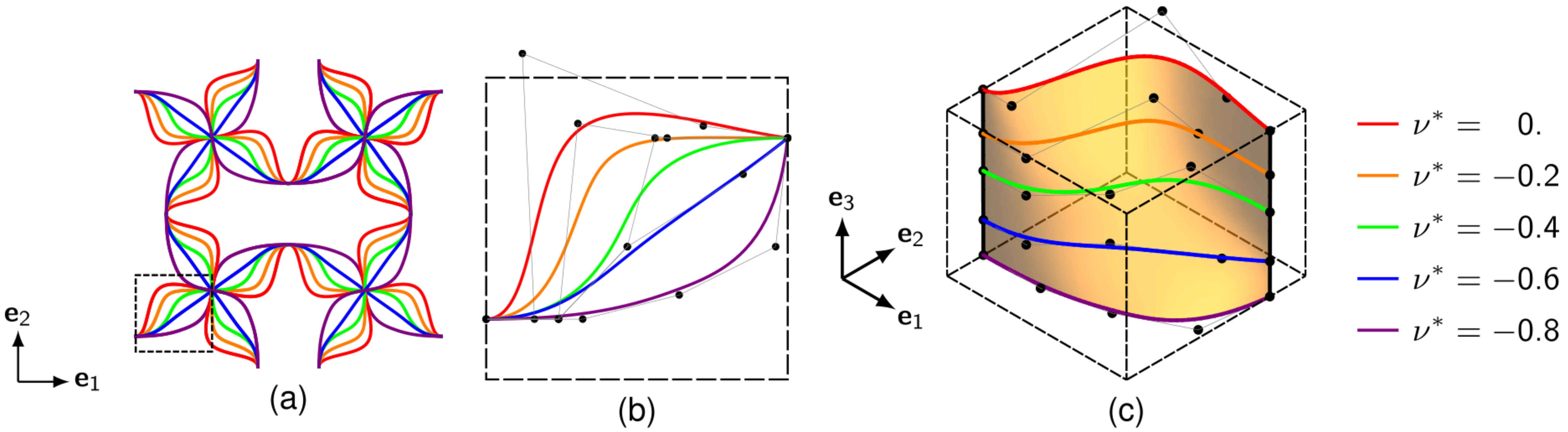
B-spline curves



(Clausen et al. *Adv. Mater* 2015)

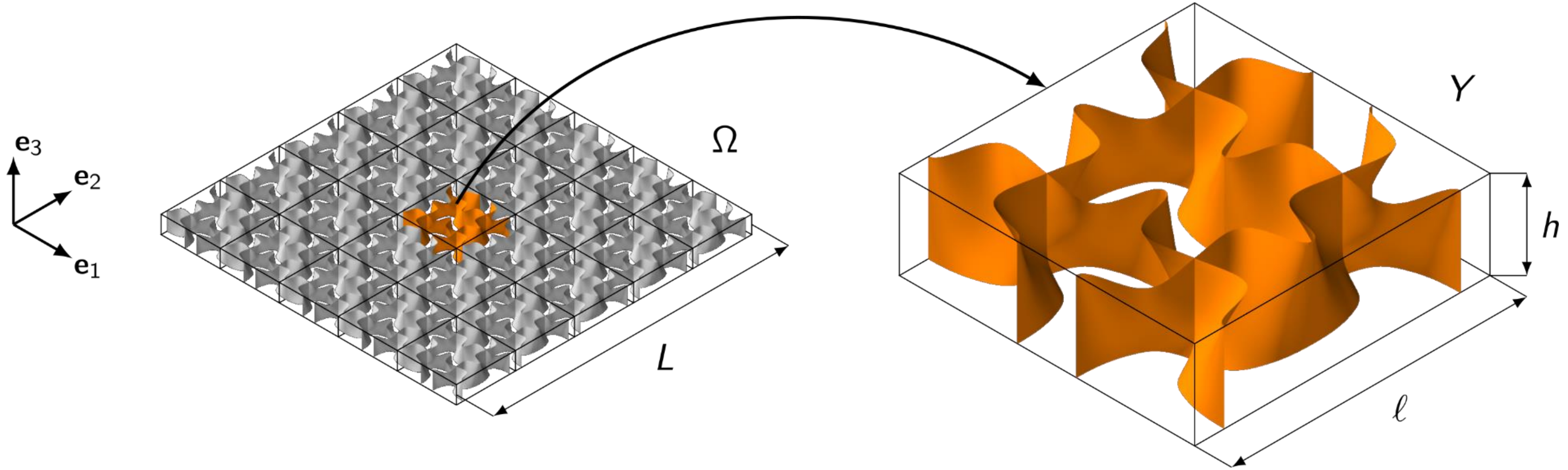


From 2D wire-based network to 3D ribbon-based lattice



B-spline surface: $$\mathbf{S}(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u)\, N_{j,q}(v)\, \mathbf{P}_{i,j}$$

Architectured panel with a ribbon based structure

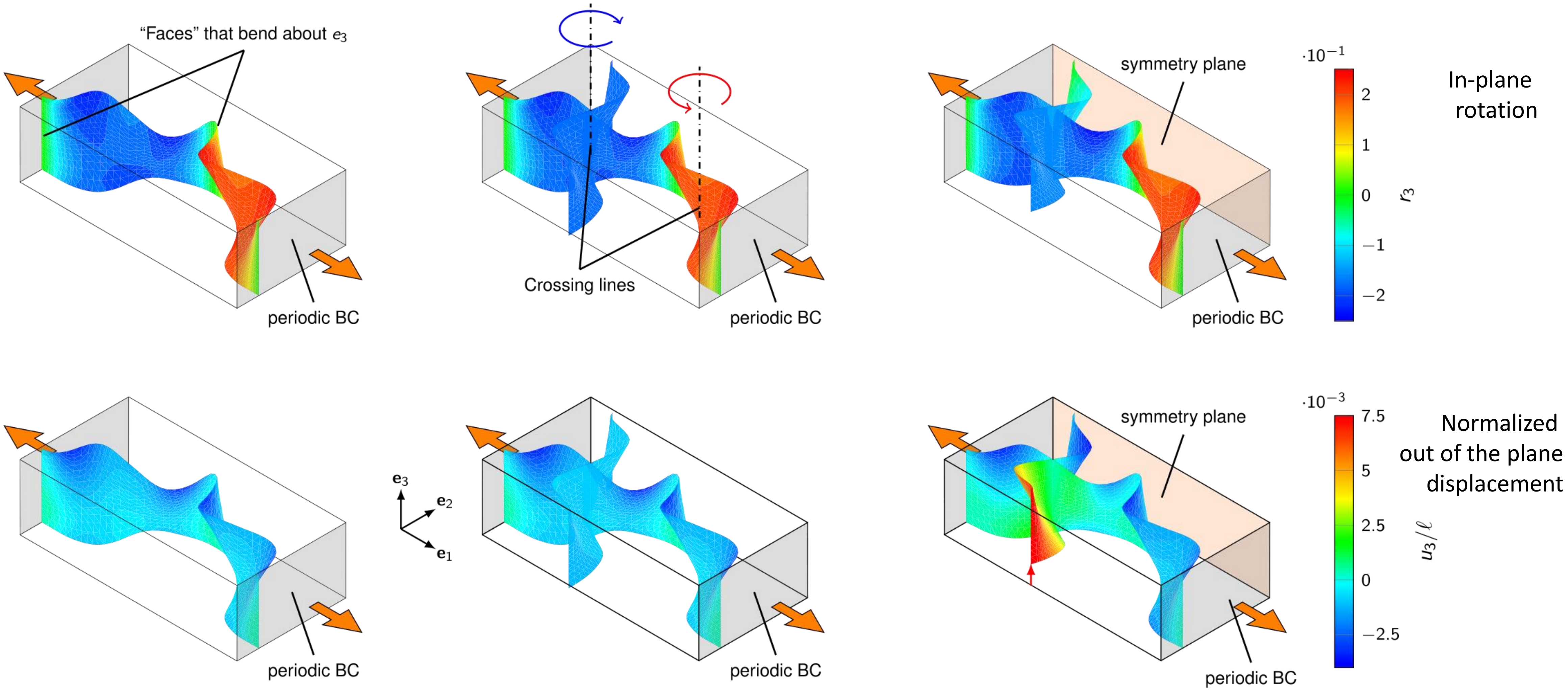


Solid Ω composed of a large number of unit cells Y

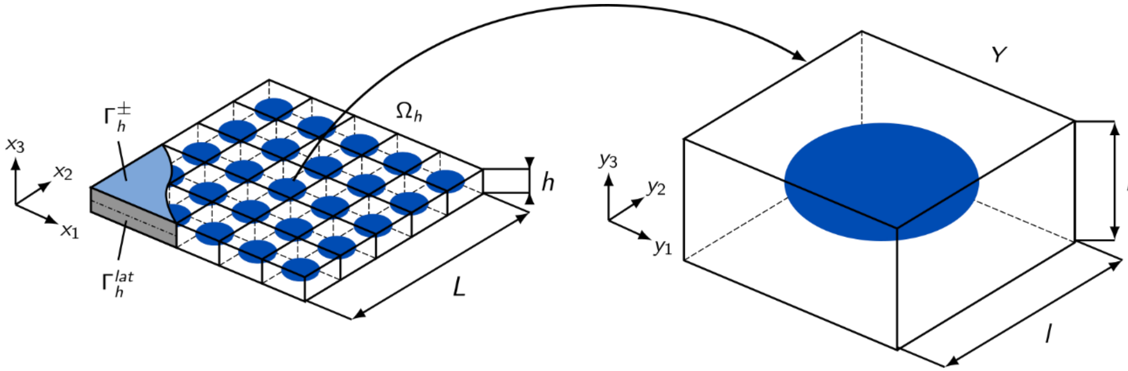
Aspect ratio: $h^\star = h/\ell$

Normalized thickness: $t^\star = t/\ell$

Mechanics of ribbons

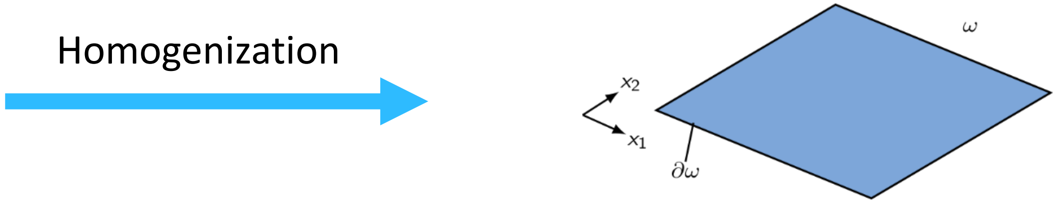


Elastic panels with a periodic micro-structure



Periodic panel

Representative unit cell



Equivalent homogeneous panel

Microscopic behavior

Linear elastic, isotropic, **heterogeneous** but **periodic**

$$\boldsymbol{\sigma}^{h\epsilon} = \mathsf{C}^{h\epsilon}(\mathbf{y}) : \varepsilon(\mathbf{u}^{h\epsilon})$$

Macroscopic behavior

Linear elastic, **orthotropic**, homogeneous

Generalized Kirchhoff–Love thin plate model

$$\begin{bmatrix} \boldsymbol{N} \\ \boldsymbol{M} \end{bmatrix} = \begin{bmatrix} \mathsf{A}^* & \mathsf{B}^* \\ {}^{\top}\mathsf{B}^* & \mathsf{D}^* \end{bmatrix} : \begin{bmatrix} \varepsilon_{\mathbf{x}}(\mathbf{U}_\alpha) \\ \chi_{\mathbf{x}}(U_3) \end{bmatrix}$$

Homogenization for thin periodic plates

- Thin plate elastic stiffness tensor

$$\begin{bmatrix} A^* & B^* \\ B^{\top *} & D^* \end{bmatrix}_{(h^\star, t^\star)} = \left[\begin{array}{ccc|ccc} A^*_{1111} & A^*_{1122} & A^*_{1112} & B^*_{1111} & B^*_{1122} & B^*_{1112} \\ A^*_{1122} & A^*_{2222} & A^*_{2212} & B^*_{2211} & B^*_{2222} & B^*_{2212} \\ A^*_{1122} & A^*_{2212} & A^*_{1212} & B^*_{1211} & B^*_{1222} & B^*_{1212} \\ \hline B^*_{1111} & B^*_{2211} & B^*_{1211} & D^*_{1111} & D^*_{1122} & D^*_{1112} \\ B^*_{1122} & B^*_{2222} & B^*_{2211} & D^*_{1122} & D^*_{2222} & D^*_{2222} \\ B^*_{1112} & B^*_{2212} & B^*_{1212} & D^*_{1112} & D^*_{2212} & D^*_{1212} \end{array}\right]$$

- Effective stiffness coefficients

$$A^*_{\alpha\beta\gamma\delta} = \frac{h}{|Y|}\int_Y \mathsf{C}(\mathbf{y}) : \left(\boldsymbol{E}^{\gamma\delta} + \varepsilon_{\mathbf{y}}(\mathbf{w}^{\gamma\delta})\right) : \left(\boldsymbol{E}^{\alpha\beta} + \varepsilon_{\mathbf{y}}(\mathbf{w}^{\alpha\beta})\right) d\mathbf{y}$$

$$B^*_{\alpha\beta\gamma\delta} = \frac{h}{|Y|}\int_Y \mathsf{C}(\mathbf{y}) : \left(y_3\boldsymbol{E}^{\gamma\delta} + \varepsilon_{\mathbf{y}}(\mathbf{p}^{\gamma\delta})\right) : \left(\boldsymbol{E}^{\alpha\beta} + \varepsilon_{\mathbf{y}}(\mathbf{w}^{\alpha\beta})\right) d\mathbf{y}$$

$$D^*_{\alpha\beta\gamma\delta} = \frac{h}{|Y|}\int_Y \mathsf{C}(\mathbf{y}) : \left(y_3\boldsymbol{E}^{\gamma\delta} + \varepsilon_{\mathbf{y}}(\mathbf{p}^{\gamma\delta})\right) : \left(y_3\boldsymbol{E}^{\alpha\beta} + \varepsilon_{\mathbf{y}ij}(\mathbf{p}^{\alpha\beta})\right) d\mathbf{y}$$

(Caillerie & Nedelec Math. Methods Appl. Sci. 1984)

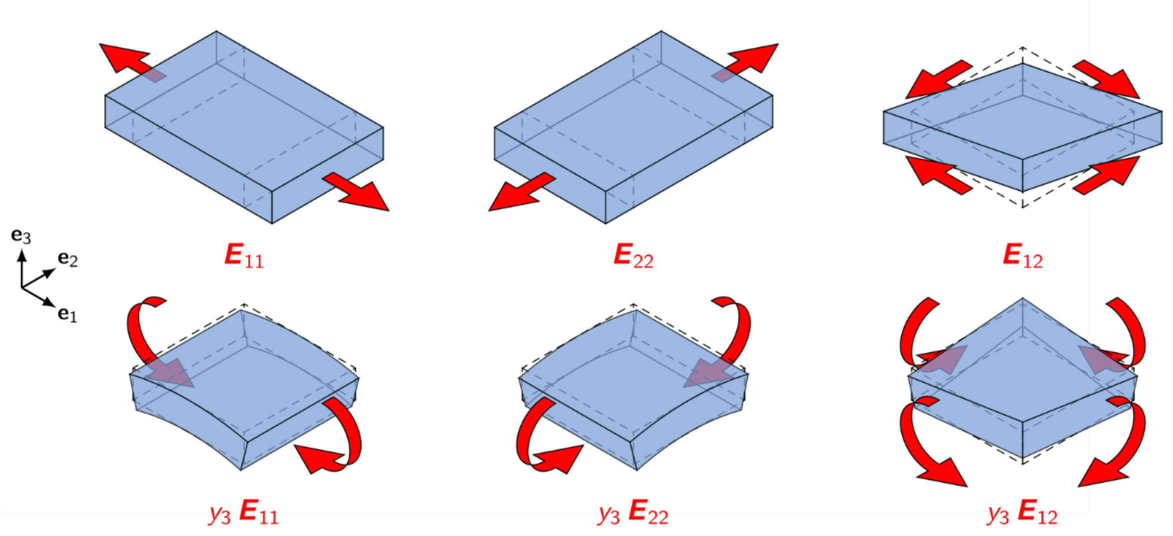
- Cell problems

$\mathbf{w}^{\gamma\delta} \in \tilde{\mathcal{V}}(Y)$ satisfy:

$$\int_Y \mathsf{C}(\mathbf{y}) : \left(\boldsymbol{E}^{\gamma\delta} + \varepsilon_{\mathbf{y}}(\mathbf{w}^{\gamma\delta})\right) : \varepsilon_{\mathbf{y}}(\mathbf{v})\, d\mathbf{y} = 0 \qquad \forall \mathbf{v} \in \tilde{\mathcal{V}}(Y)$$

$\mathbf{p}^{\gamma\delta} \in \tilde{\mathcal{V}}(Y)$ satisfy:

$$\int_Y \mathsf{C}(\mathbf{y}) : \left(y_3\, \boldsymbol{E}^{\gamma\delta} + \varepsilon_{\mathbf{y}}(\mathbf{p}^{\gamma\delta})\right) : \varepsilon_{\mathbf{y}}(\mathbf{v})\, d\mathbf{y} = 0 \qquad \forall \mathbf{v} \in \tilde{\mathcal{V}}(Y)$$



Results interpretation: stiffness C* vs. compliance S*

- **Hooke's law**

$$\begin{bmatrix} \boldsymbol{N} \\ \boldsymbol{M} \end{bmatrix} = [\mathsf{C}^*] : \begin{bmatrix} \varepsilon_{\mathbf{x}}(\mathbf{U}_\alpha) \\ \chi_{\mathbf{x}}(U_3) \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_{\mathbf{x}}(\mathbf{U}_\alpha) \\ \chi_{\mathbf{x}}(U_3) \end{bmatrix} = [\mathsf{S}^*] : \begin{bmatrix} \boldsymbol{N} \\ \boldsymbol{M} \end{bmatrix}$$

$$\mathsf{C}^* = \begin{bmatrix} \mathsf{A}^* & \mathsf{B}^* \\ {}^\top\mathsf{B}^* & \mathsf{D}^* \end{bmatrix} = \left[\begin{array}{ccc|ccc} A^*_{1111} & A^*_{1122} & 0 & B^*_{1111} & B^*_{1122} & 0 \\ A^*_{1122} & A^*_{2222} & 0 & B^*_{2211} & B^*_{2222} & 0 \\ 0 & 0 & A^*_{1212} & 0 & 0 & B^*_{1212} \\ \hline B^*_{1111} & B^*_{2211} & 0 & D^*_{1111} & D^*_{1122} & 0 \\ B^*_{1122} & B^*_{2222} & 0 & D^*_{1122} & D^*_{2222} & 0 \\ 0 & 0 & B^*_{1212} & 0 & 0 & D^*_{1212} \end{array}\right]$$

$$\mathsf{S}^* = (\mathsf{C}^*)^{-1} = \left[\begin{array}{ccc|ccc} S^*_{1111} & S^*_{1122} & 0 & S^*_{1144} & S^*_{1155} & 0 \\ S^*_{1122} & S^*_{2222} & 0 & S^*_{2244} & S^*_{2255} & 0 \\ 0 & 0 & S^*_{1212} & 0 & 0 & S^*_{1245} \\ \hline S^*_{1144} & S^*_{2244} & 0 & S^*_{4444} & S^*_{4455} & 0 \\ S^*_{1155} & S^*_{2255} & 0 & S^*_{4455} & S^*_{5555} & 0 \\ 0 & 0 & S^*_{1245} & 0 & 0 & S^*_{4545} \end{array}\right]$$

- **Material coefficients (stiffness)**

$$E^* = \frac{1}{h} A^*_{1111} \left(1 - \left(\frac{A^*_{1122}}{A^*_{1111}}\right)\right)$$

$$\nu^* = \frac{A^*_{1122}}{A^*_{1111}}$$

- **Material coefficients (compliance)**

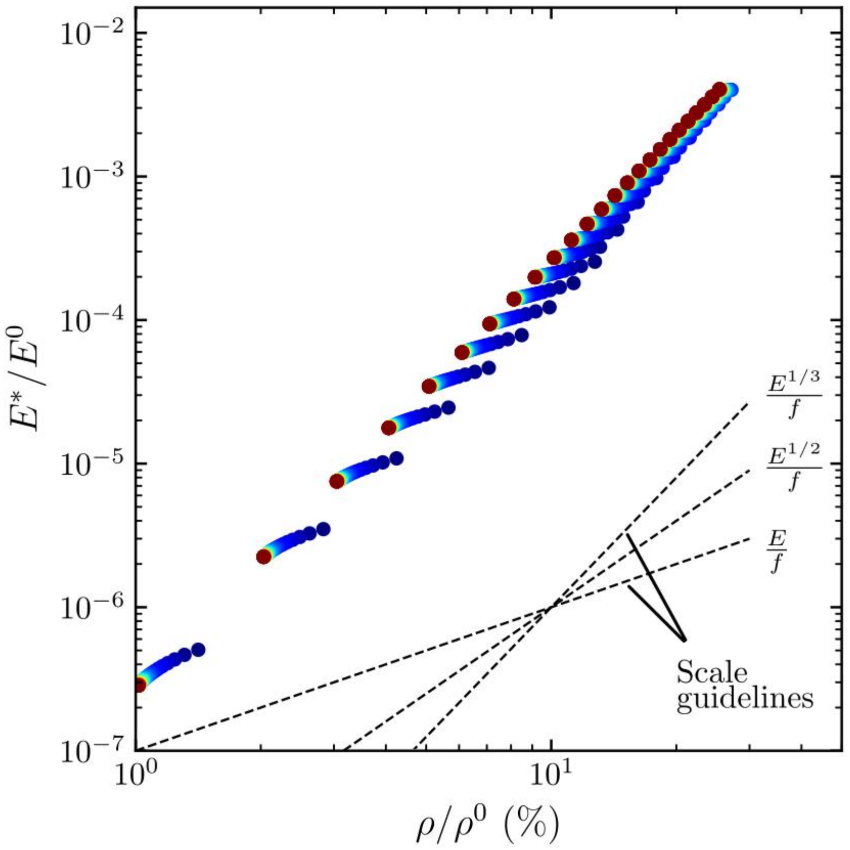
$$E^* = \frac{1}{h^\star\, S_{1111}} \qquad \gamma^* = \frac{S_{1155}}{S_{1111}}$$

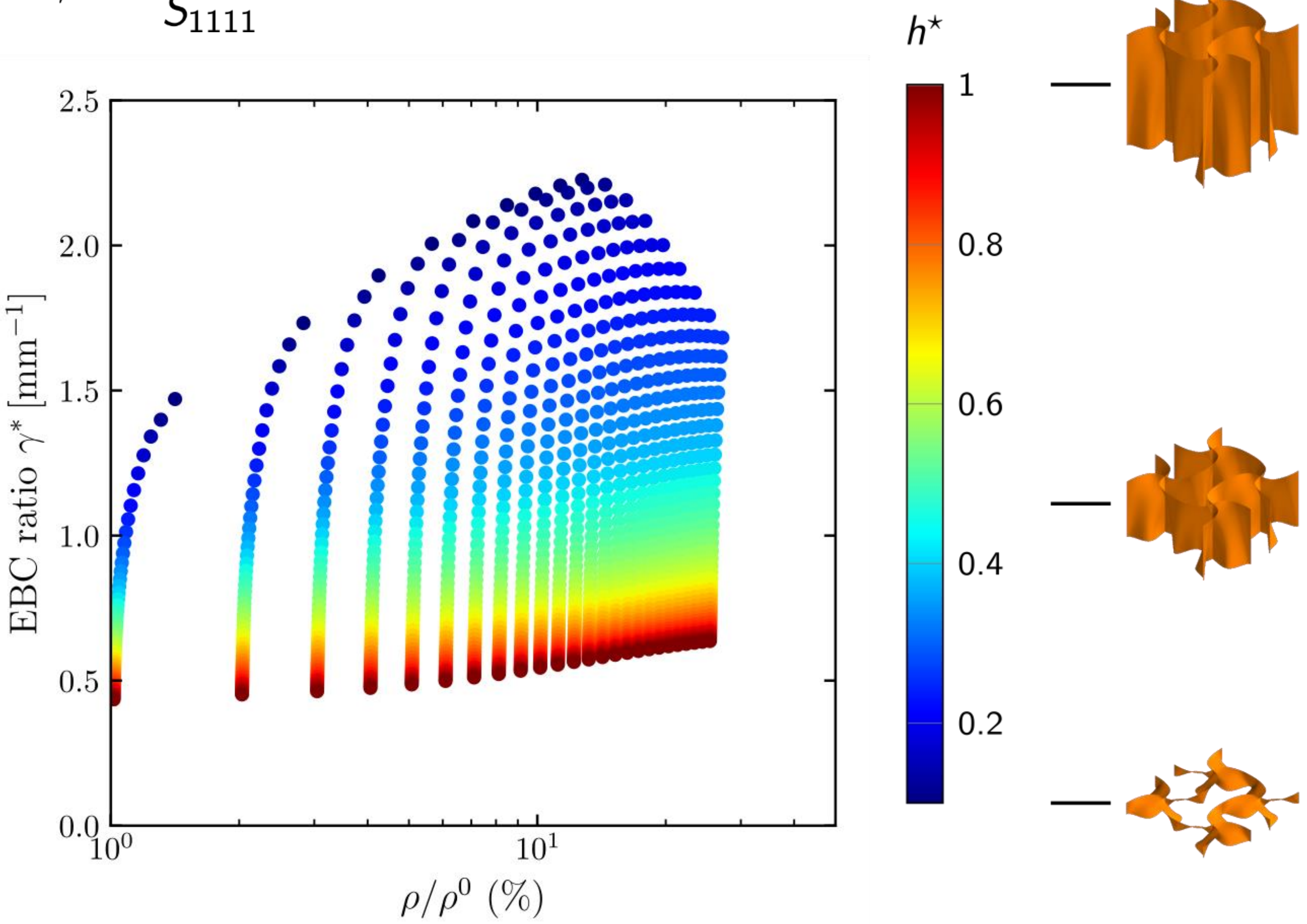
$$\nu^* = -\frac{S^*_{1122}}{S^*_{1111}}$$

Parametric analysis using compliance tensor S

$$\mathsf{S} = \mathsf{C}^{-1} \qquad E^* = \frac{1}{h^\star\, S_{1111}} \qquad \gamma^* = \frac{S_{1155}}{S_{1111}}$$

$t^\star < h/5$

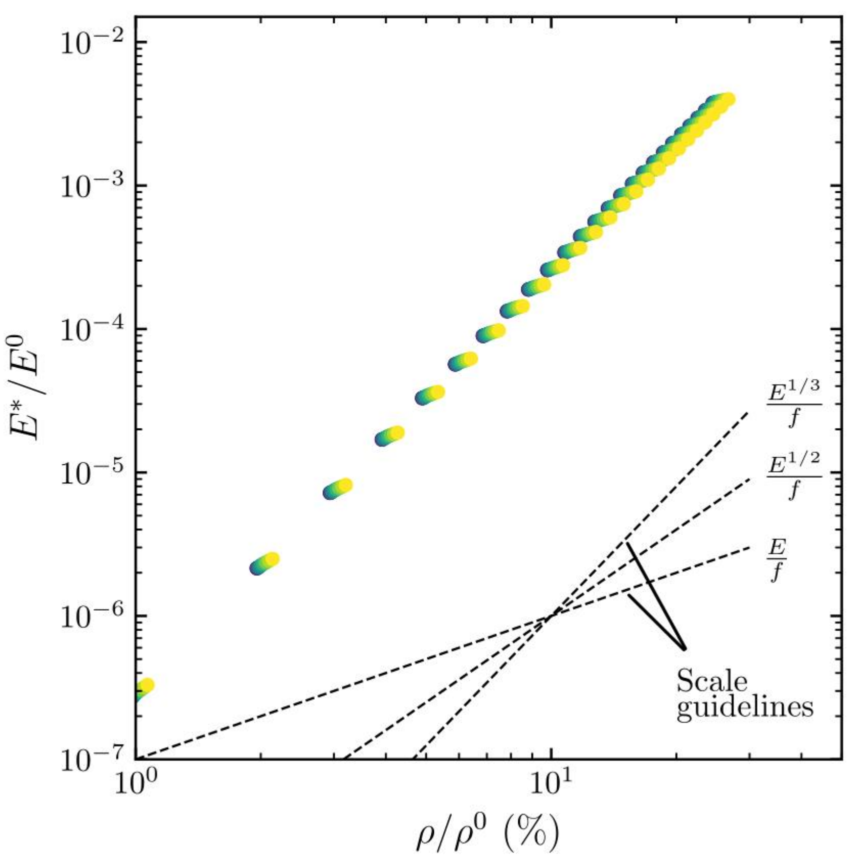


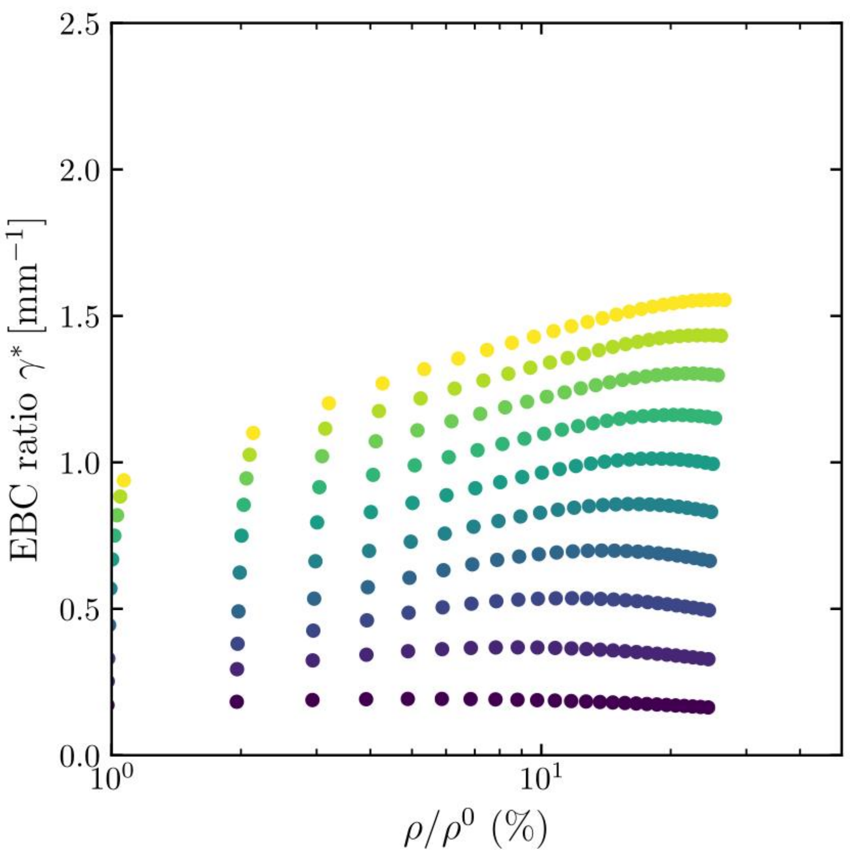


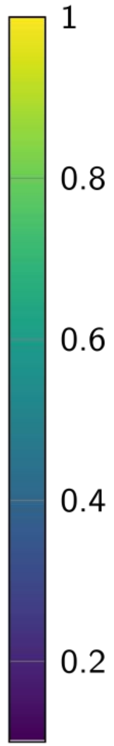
Parametric analysis using compliance tensor S

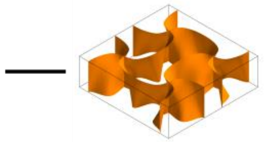
$$\mathsf{S} = \mathsf{C}^{-1} \qquad E^* = \frac{1}{h^\star\, S_{1111}} \qquad \gamma^* = \frac{S_{1155}}{S_{1111}}$$

$h^\star = 0.3$



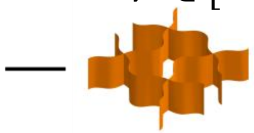




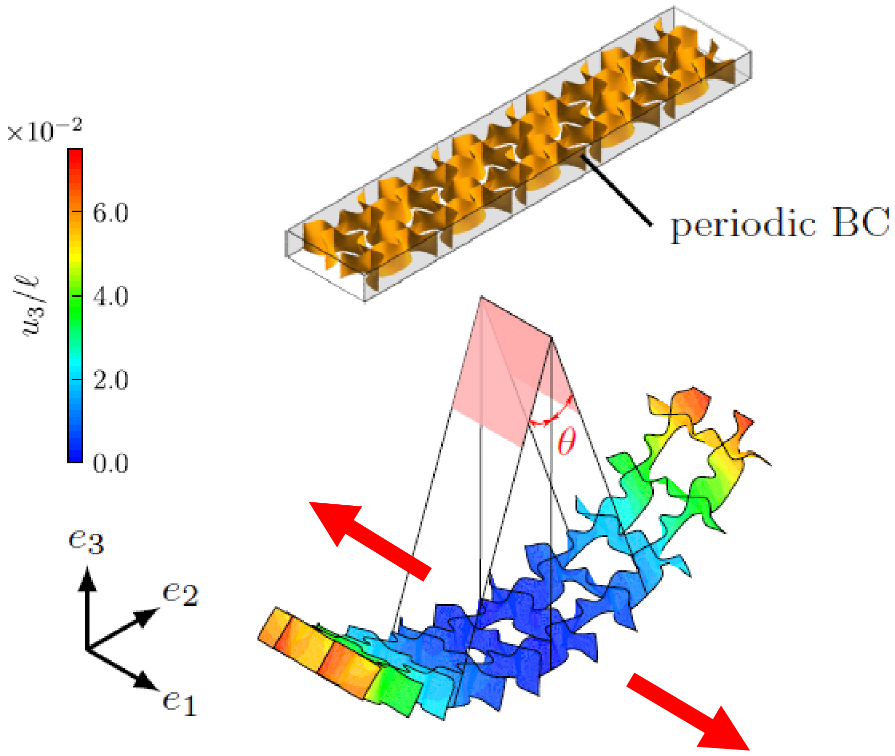


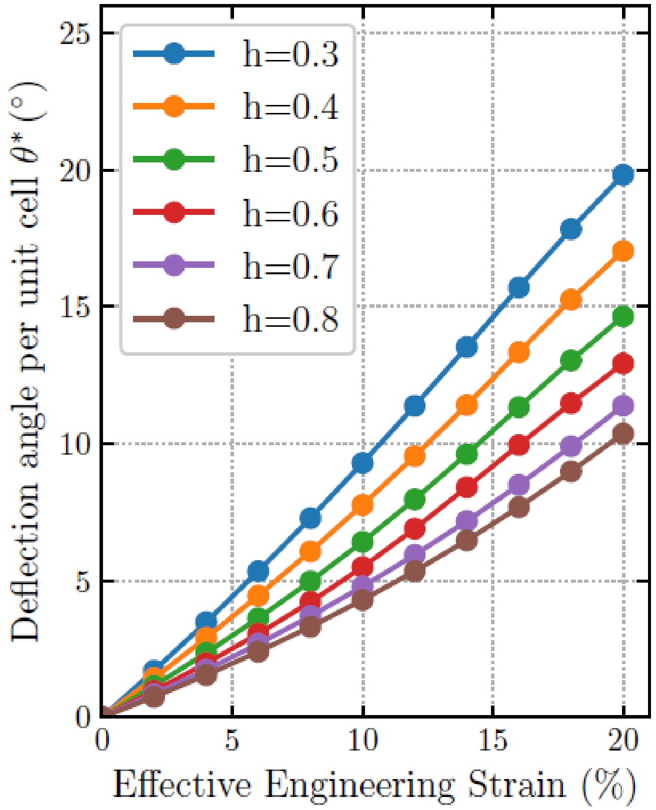
$\nu^* \in [-0.8,0]$

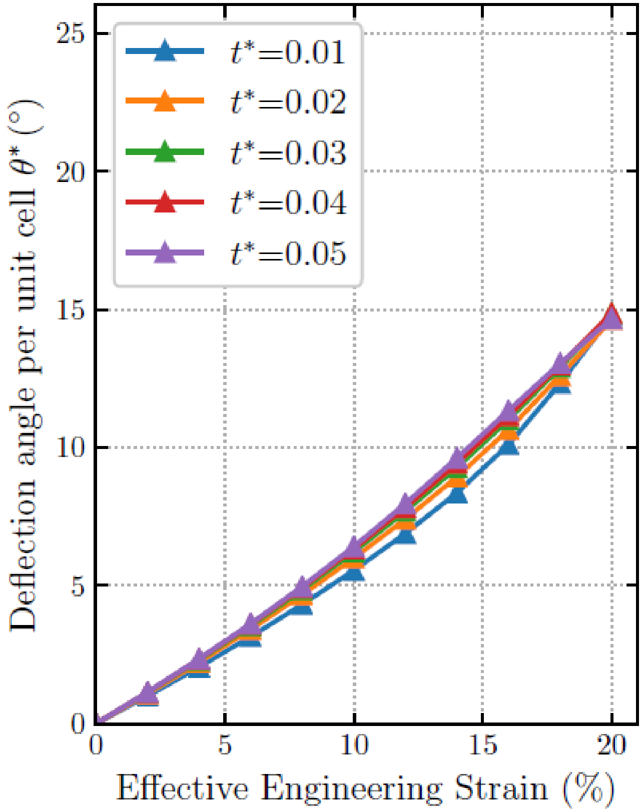
$\nu^* \in [-0.5,-0{,}3]$



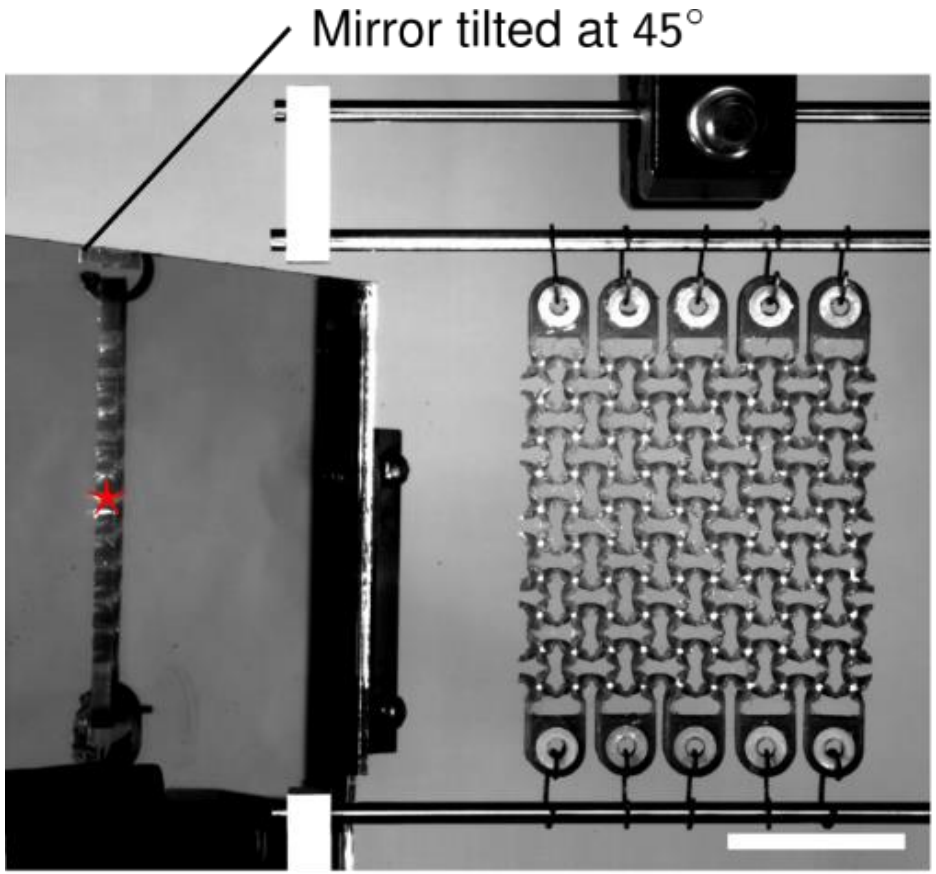
Characterization at finite strain

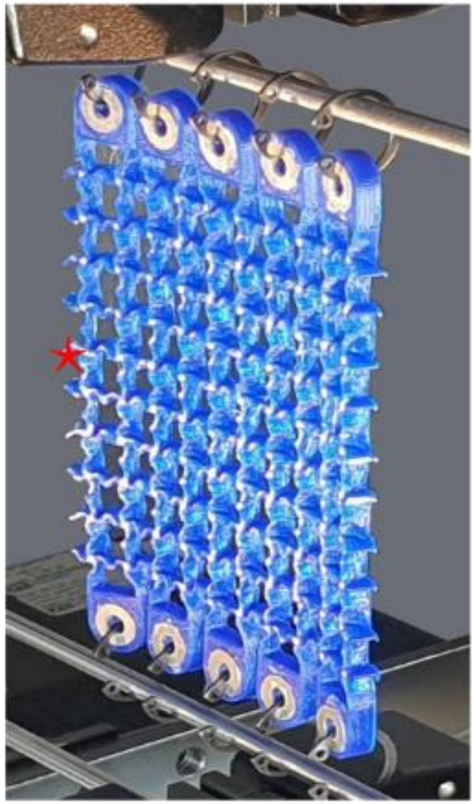


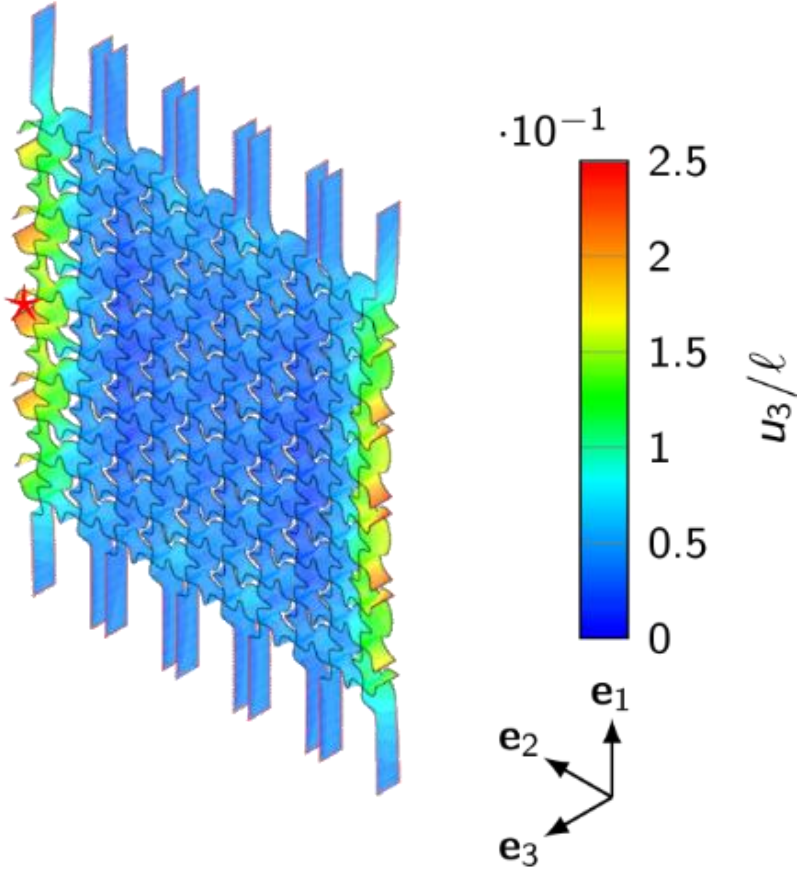




Experimental characterization – Tensile testing

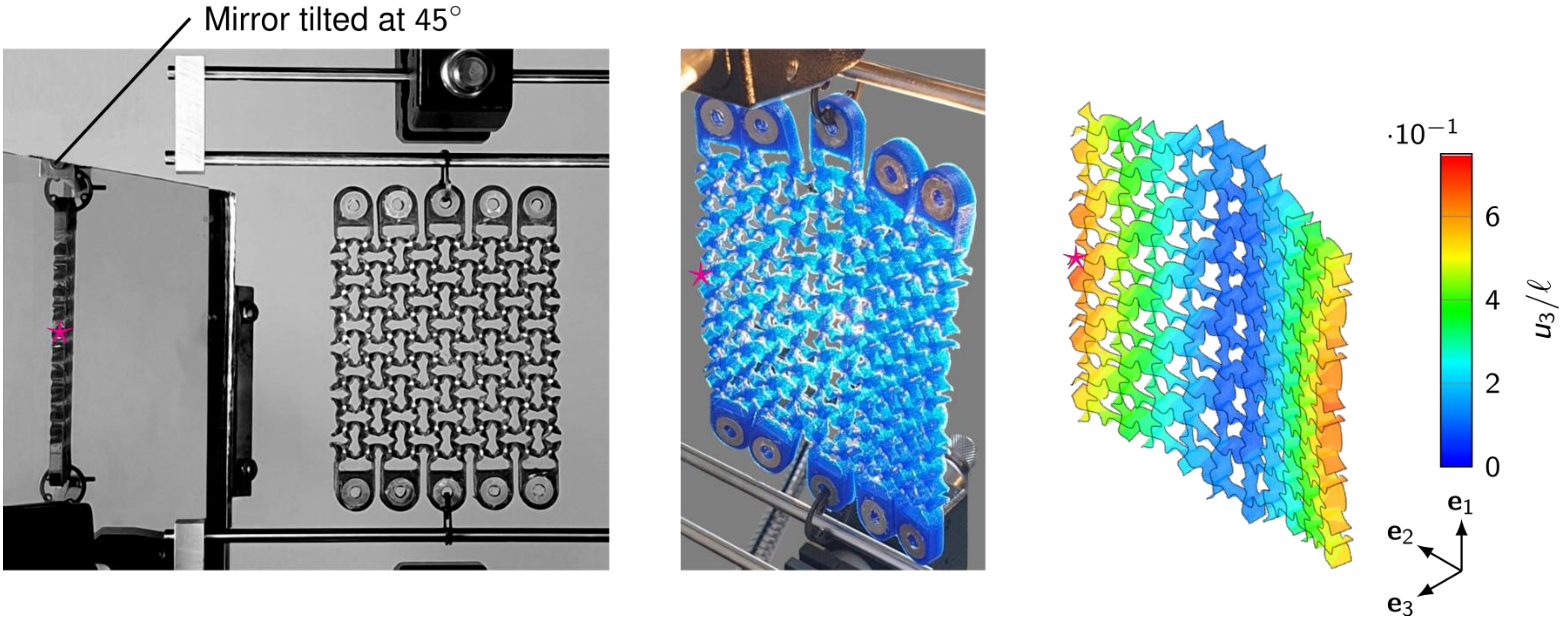




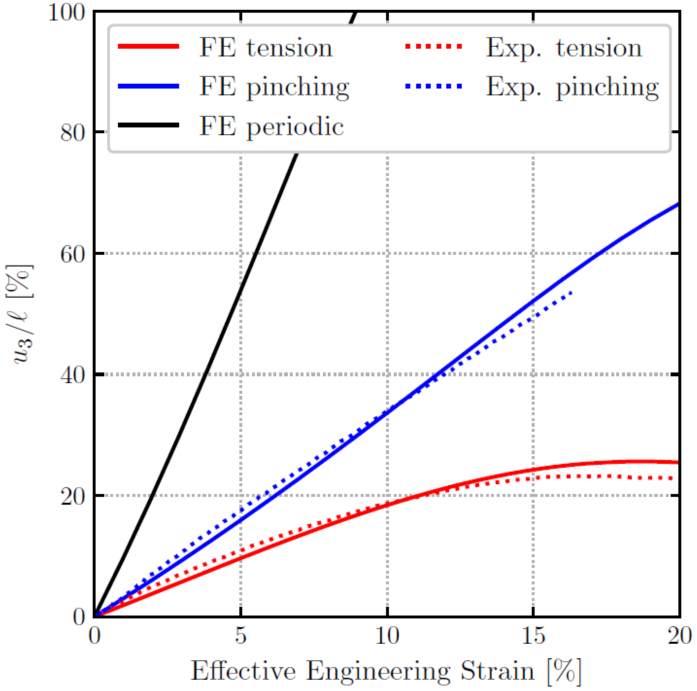


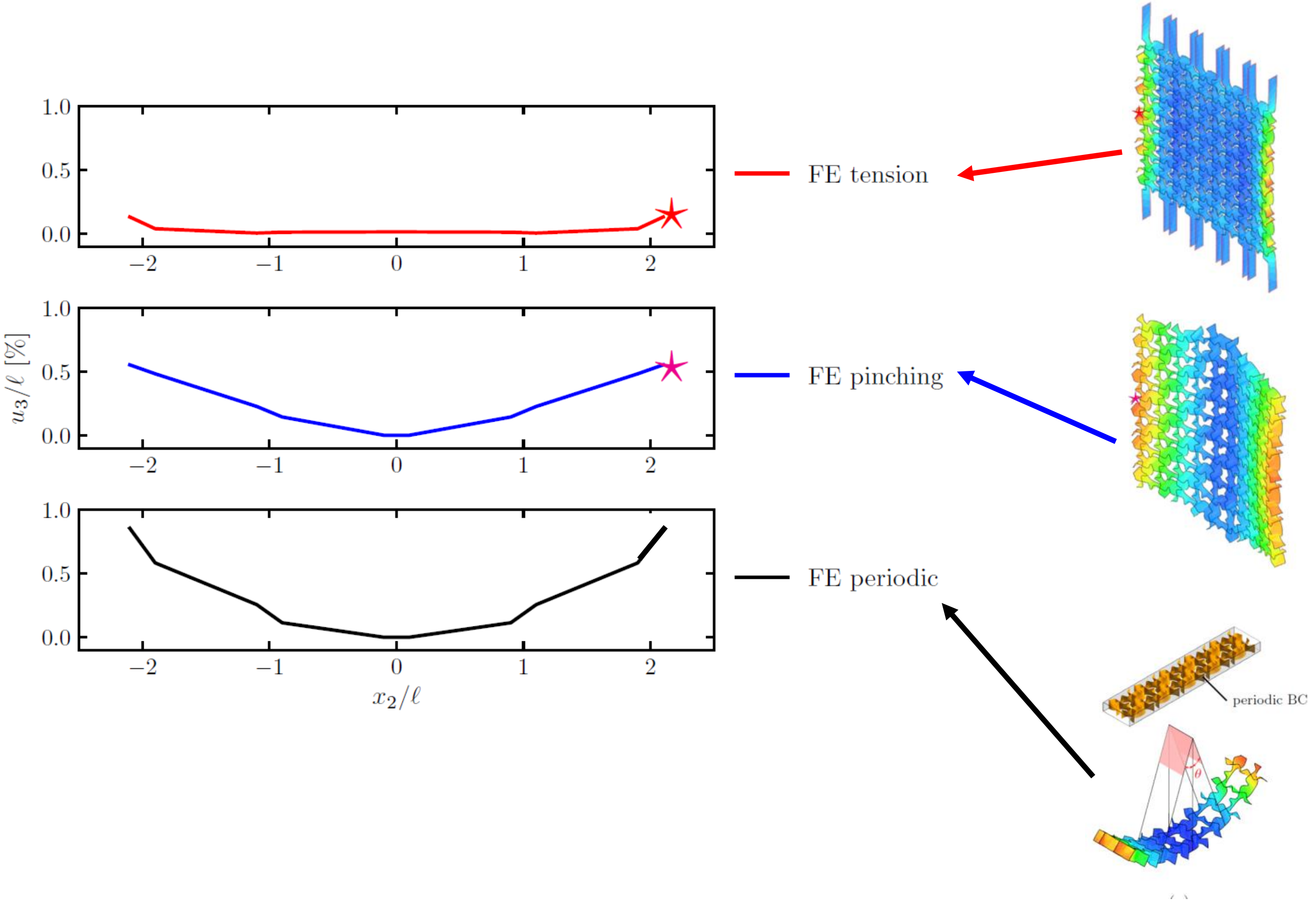
3D printed FFF – TPU (credit: Drahi-X)

Experimental characterization – Pinching testing



Out of the plane displacement





Ribbon-based panels: summary

Mechanics of ribbon-based networks

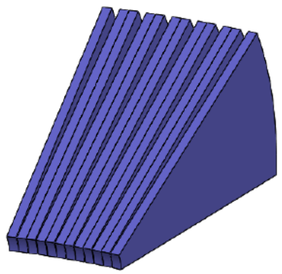
- Architecture for extension-bending coupling
- Parametric analysis at small strain and finite strain
- Testing conditions

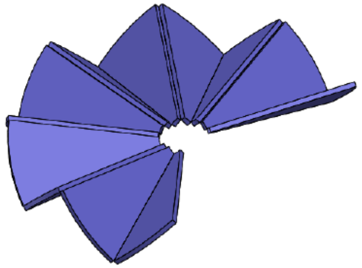
Future developments

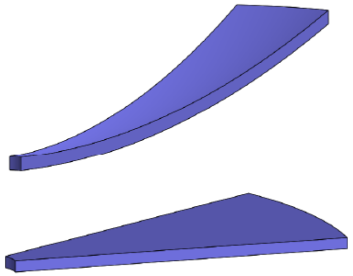
- Gradient of material properties, break the periodicity
- Actuation through multi-physics coupling

F. Agnelli, M. Tricarico, A. Constantinescu. “Shape-shifting panel from 3D printed undulated ribbon lattice”. *Extreme Mech. Lett.* 42 (2021)

Perspectives







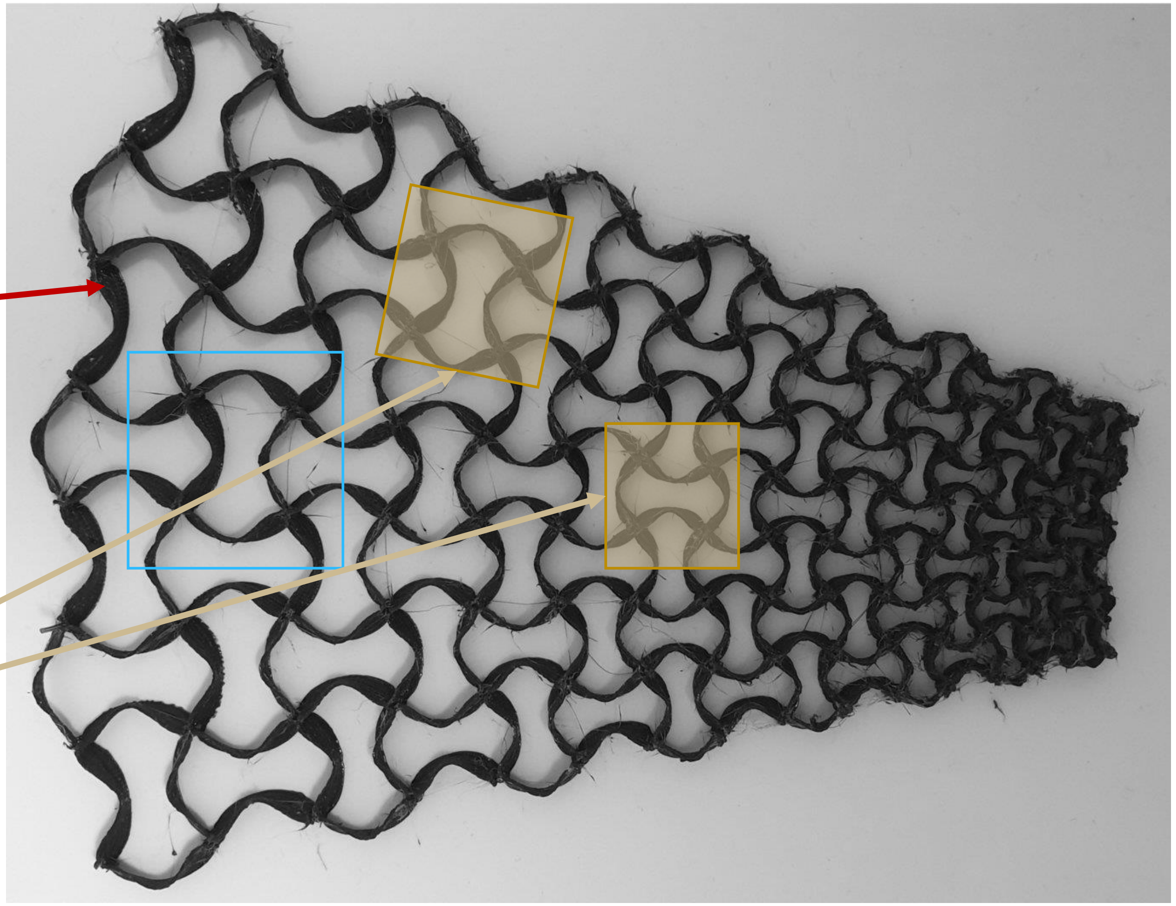
Variable pattern along the height

- Continuous or discontinuous design along the height

Loss of periodicity:

- vary the design along the unit cells

Embedding responsive materials to the design



Internship project: Jianzhou MA (ENSTA)

Outline

- **Mechanical characterization of periodic panels**
  - Analysis of a ribbon-based periodic panel
    - Asymptotic homogenization theory
    - Generalized Kirchhoff-Love model

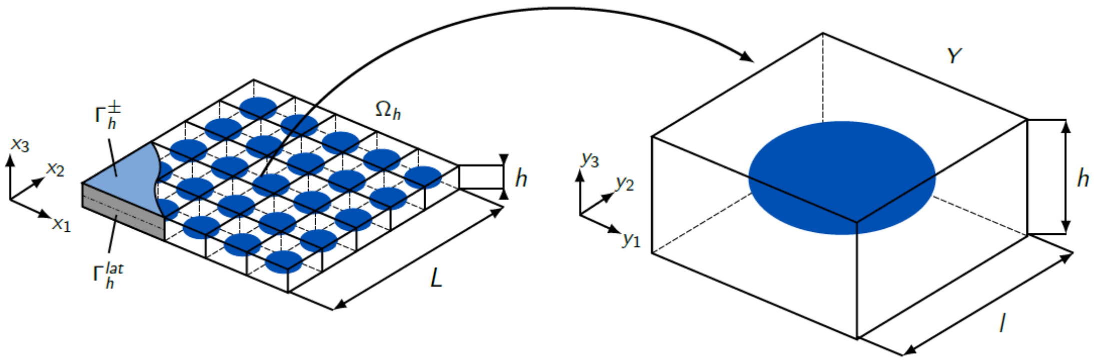
- **Topology optimization using a level set method**
  - Finding an elementary form associated to a target elastic stiffness
  - Shape derivative in the sense of Hadamard
  - Classical level set method

"*The art of structure is where to put the holes*"

Robert Le Ricolais, 1894-1977

Tailoring elastic response

- Representative unit cell



- A*, B* and D* obtained by asymptotic homogenization

- Generalized Kirchhoff-Love behavior for thin plates

$$\boldsymbol{C}^{\text{target}} = \left[\begin{array}{ccc|ccc} A^*_{1111} & A^*_{1122} & \star & B^*_{1111} & B^*_{1122} & \star \\ A^*_{1122} & A^*_{2222} & \star & B^*_{2211} & B^*_{2222} & \star \\ \star & \star & \star & \star & \star & \star \\ \hline B^*_{1111} & B^*_{2211} & \star & D^*_{1111} & D^*_{1122} & \star \\ B^*_{1122} & B^*_{2222} & \star & D^*_{1122} & D^*_{2222} & \star \\ \star & \star & \star & \star & \star & \star \end{array}\right]$$

Optimization problem

The shape $S \subset Y$ is a smooth, open, bounded subset

$$\mathcal{J}(S) = \frac{1}{2}||\mathbf{A}^* - \mathbf{A}^{\text{target}}||^2_{\eta_A} + \frac{1}{2}||\mathbf{B}^* - \mathbf{B}^{\text{target}}||^2_{\eta_B} + \frac{1}{2}||\mathbf{D}^* - \mathbf{D}^{\text{target}}||^2_{\eta_D}$$

- $\|\ \|$: weighted Euclidean norm $\quad \eta$: weights coefficients
- $\mathrm{A}^{\text{target}}$, $\mathrm{B}^{\text{target}}$ and $\mathrm{D}^{\text{target}}$: target elastic tensor values

Optimization problem

$$\inf_{S \subset \mathcal{U}_{ad}} \mathcal{J}(S)$$

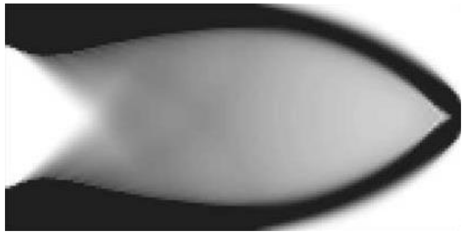
$$\mathbf{w}^{\gamma\delta} \in \tilde{\mathcal{V}}(Y) \text{ satisfy: } \int_Y C_{ijpq} \left(\delta_{p\gamma}\delta_{q\delta} + \varepsilon_{\mathbf{y}pq}(\mathbf{w}^{\gamma\delta})\right) \varepsilon_{\mathbf{y}ij}(\mathbf{v})\, d\mathbf{y} = 0 \qquad \forall \mathbf{v} \in \tilde{\mathcal{V}}(Y)$$

$$\mathbf{p}^{\gamma\delta} \in \tilde{\mathcal{V}}(Y) \text{ satisfy: } \int_Y C_{ijpq} \left(y_3\delta_{p\gamma}\delta_{q\delta} + \varepsilon_{\mathbf{y}pq}(\mathbf{p}^{\gamma\delta})\right) \varepsilon_{\mathbf{y}ij}(\mathbf{v})\, d\mathbf{y} = 0 \qquad \forall \mathbf{v} \in \tilde{\mathcal{V}}(Y)$$

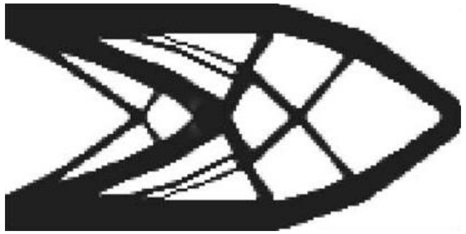
Topology optimization

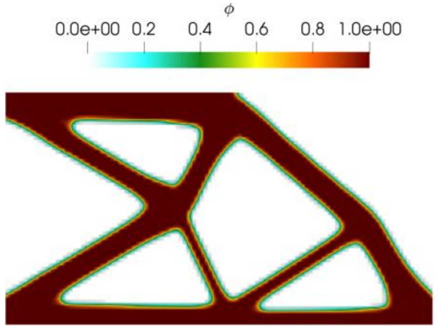
- **Density based methods**
  - Homogenization method
    - Pioneering work in (Bendsøe & Kikuchi *CMAME* 1988)
  - Solid Isotropic Material with Penalization (SIMP) algorithm
    - (Bendsøe *Struct. Opt.* 1989)

- **Implicit method**
  - Level set method
    - (Allaire *C. R. Math.* 2002)
    - (Wang, Mei & Wang *JMPS* 2003)
  - Phase-field
    - (Takezawa et al. *J. Comp. Phys.* 2010)

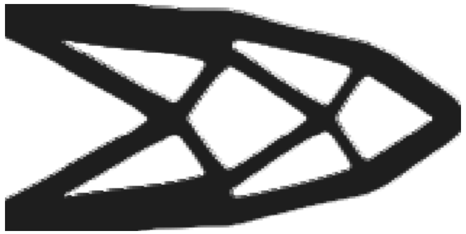


(Allaire et al. *J. Comp. Phys.* 2004)





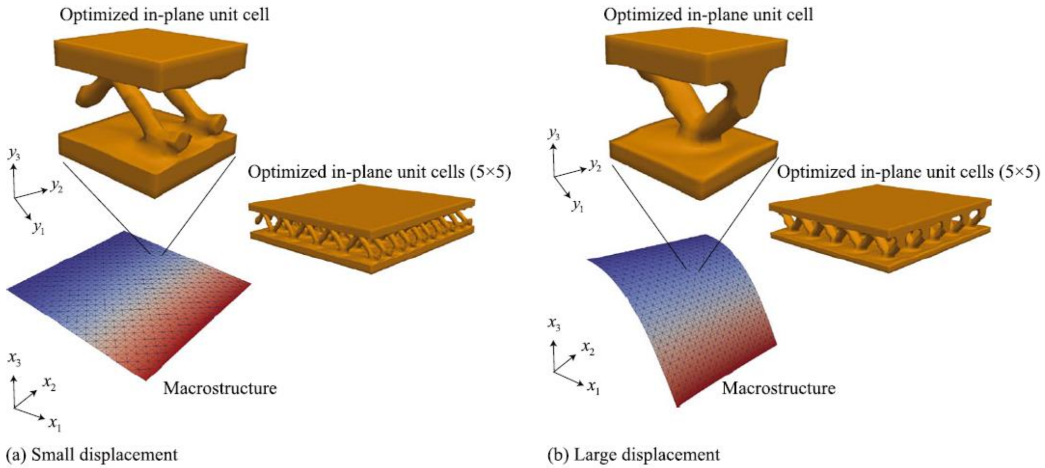
(Carraturo et al. 2019 *Comp. Mech.* 2019)



(Allaire et al. *J. Comp. Phys.* 2004)

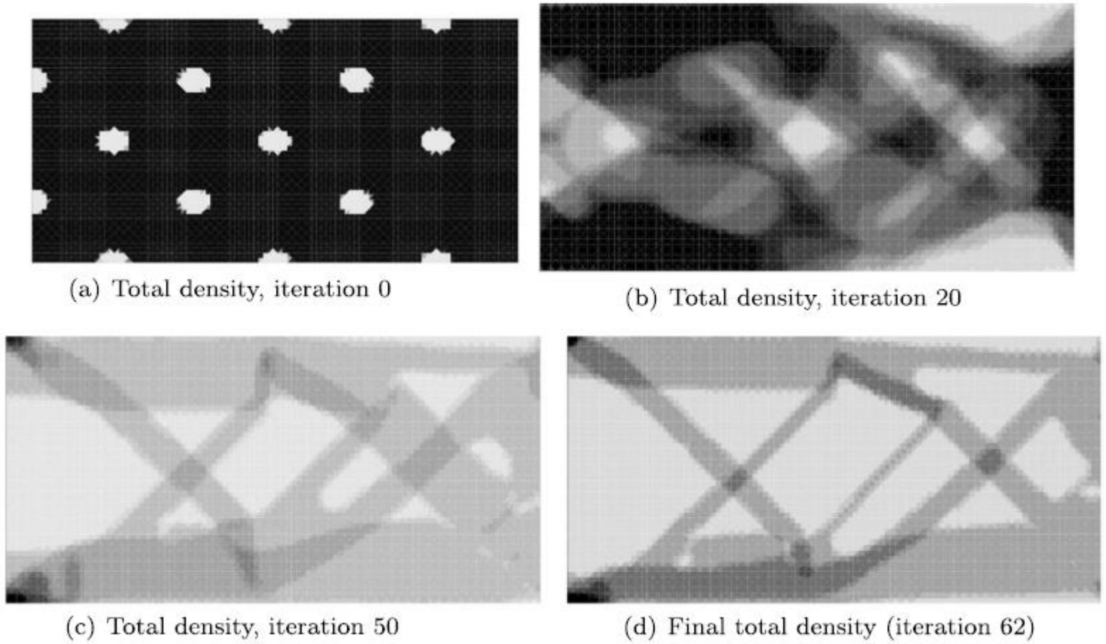
Topology optimization for panels

- **Density based methods**
  - Solid Isotropic Material with Penalization (SIMP) algorithm



(Nishi et al. *Int. J. Numer. Methods Eng.* 2017)

- **Implicit method**
  - Level set method



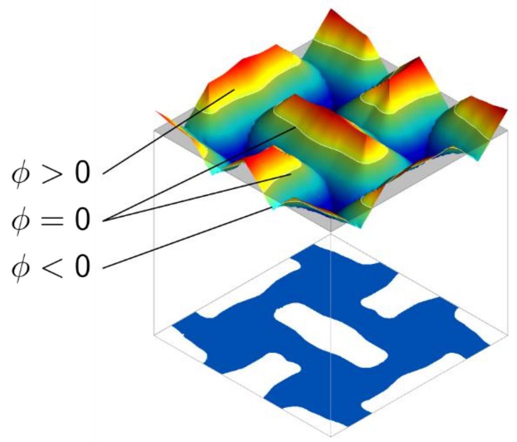
(Allaire & Delgado *J. Mech. Phys. Solids* 2016)

Topology optimization using a level set method

- Level set function representing the material distribution

(Osher & Sethian *J. Comp. Phy.* 1988)

$$\begin{cases} \phi(\mathbf{y}) < 0 & \text{if} \quad \mathbf{y} \in S & \text{(material)} \\ \phi(\mathbf{y}) = 0 & \text{if} \quad \mathbf{y} \in \partial S & \text{(boundary)} \\ \phi(\mathbf{y}) > 0 & \text{if} \quad \mathbf{y} \in Y \backslash S & \text{(void)} \end{cases}$$



- Boundary evolution
  - Introduce a pseudo time $t \in \mathbb{R}^+$ to characterize the evolution of *S(t)*
  - Differentiating $\phi(\boldsymbol{y}(t), t) = 0 \qquad \forall \boldsymbol{y}(t) \in \partial S(t)$

$$\frac{\partial \phi}{\partial t}(\mathbf{y}, t) + V(\mathbf{y}, t)\,|\nabla \phi(\mathbf{y}, t)| = 0 \qquad \forall t,\ \forall \mathbf{y} \in Y$$

known as a *Hamilton-Jacobi equation*

- Smooth inter-phase approach

(Allaire, Dapogny, Delgado & Michailidis *ESAIM: COCV* 2014)

$$\mathrm{C}^e(\mathbf{x}) = \mathcal{H}_e(\phi)(\mathrm{C}^{\bar{S}} - \mathrm{C}^S) + \mathrm{C}^S$$

$$\mathcal{H}_e(t) = \begin{cases} 0 & \text{if } t < -e \\ \frac{1}{2}\left(1 + \frac{t}{e} + \frac{1}{\pi}\sin\left(\frac{\pi t}{e}\right)\right) & \text{if } |t| \le e \\ 1 & \text{if } t > e \end{cases}$$

The shape derivative in the sense of Hadamard

- The objective function is shape differentiable and admits a derivative at $\boldsymbol{\theta} = \mathbf{0}$ which is

$$\mathcal{J}'(S)(\theta) = -\int_{\Gamma^S} \theta \cdot \mathbf{n} \left(f_A(\mathbf{s}) + f_B(\mathbf{s}) + f_D(\mathbf{s})\right) \, d\mathbf{s}$$

where

$$f_A(\mathbf{s}) = \frac{r}{|Y|} \eta^A_{\alpha\beta\gamma\delta} \left(A^*_{\alpha\beta\gamma\delta}(d_S) - A^{\text{target}}_{\alpha\beta\gamma\delta}\right) \left(\mathbf{E}^{\gamma\delta} + \varepsilon_{\mathbf{y}}(\mathbf{w}^{\gamma\delta})\right) : \left(\mathsf{C}^{\bar{S}} - \mathsf{C}^S\right) : \left(\mathbf{E}^{\alpha\beta} + \varepsilon_{\mathbf{y}}(\mathbf{w}^{\alpha\beta})\right)$$

$$f_B(\mathbf{s}) = \frac{r}{|Y|} \eta^B_{\alpha\beta\gamma\delta} \left(B^*_{\alpha\beta\gamma\delta}(d_S) - B^{\text{target}}_{\alpha\beta\gamma\delta}\right) \left(\mathbf{E}^{\gamma\delta} + \varepsilon_{\mathbf{y}}(\mathbf{w}^{\gamma\delta})\right) : \left(\mathsf{C}^{\bar{S}} - \mathsf{C}^S\right) : \left(\mathbf{P}^{\alpha\beta} + \varepsilon_{\mathbf{y}}(\mathbf{p}^{\alpha\beta})\right)$$

$$f_D(\mathbf{s}) = \frac{r}{|Y|} \eta^D_{\alpha\beta\gamma\delta} \left(D^*_{\alpha\beta\gamma\delta}(d_S) - D^{\text{target}}_{\alpha\beta\gamma\delta}\right) \left(\mathbf{P}^{\gamma\delta} + \varepsilon_{\mathbf{y}}(\mathbf{p}^{\gamma\delta})\right) : \left(\mathsf{C}^{\bar{S}} - \mathsf{C}^S\right) : \left(\mathbf{P}^{\alpha\beta} + \varepsilon_{\mathbf{y}}(\mathbf{p}^{\alpha\beta})\right)$$

Hence, a descent direction can always be selected by choosing

$$\theta = \left(f_A(\mathbf{s}) + f_B(\mathbf{s}) + f_D(\mathbf{s})\right) \mathbf{n}.$$

(Allaire, Dapogny, Delgado & Michailidis *ESAIM: COCV* 2014)

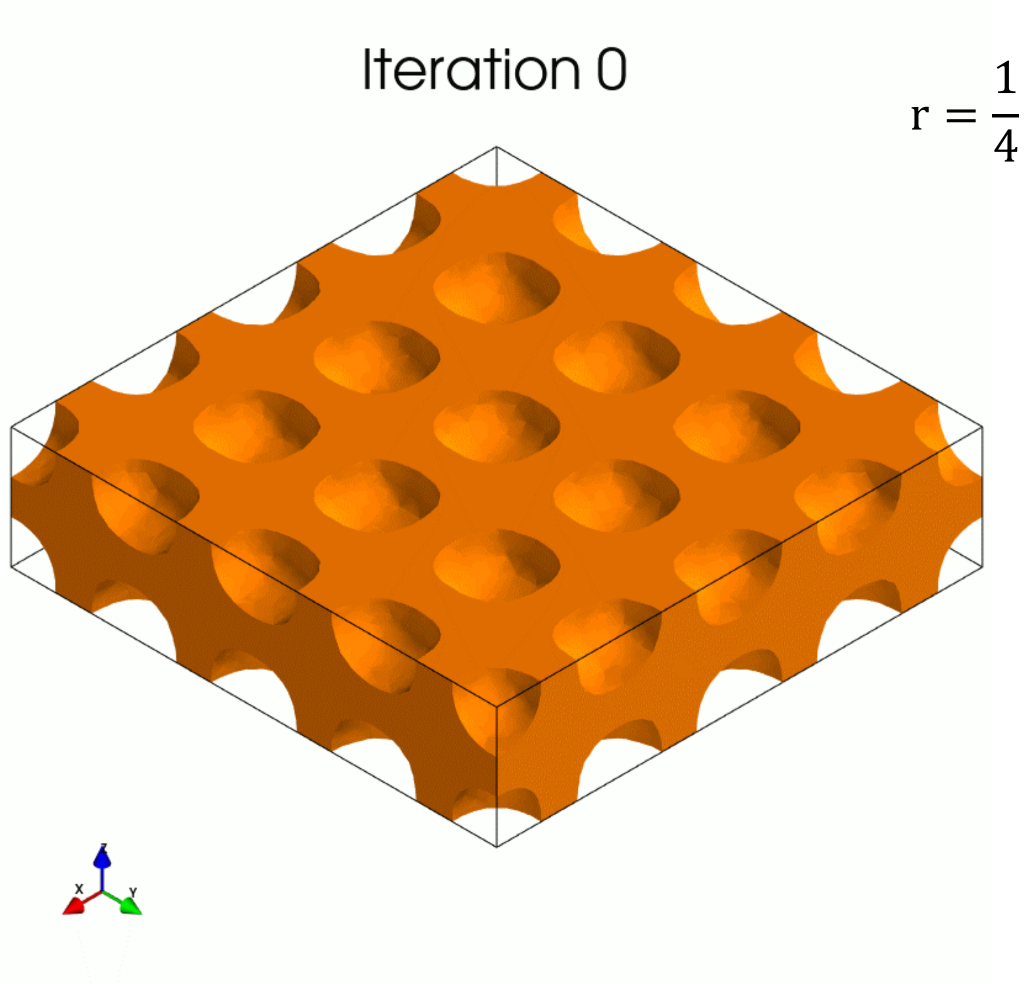
Numerical algorithm

- Initialize the level set ϕ_0 corresponding to the initial shape S_0
- Iterate until convergence for k ≥ 0
  - Re-initialize ϕ_0 into the signed distance function d_{S_0}
  - Calculate the local solutions $\mathbf{w}^{\gamma\delta}$ and $\mathbf{p}^{\gamma\delta}$ by solving the cell problems in Y
  - Compute the shape gradient $J(S^k)(\boldsymbol{\theta}^k)$ for the domain S^k
  - Deform the domain S_k by solving the above Hamilton-Jacobi equation.

    // The volume fraction should remain bounded in a given set

    // S_{k+1} is characterized by the level set ϕ_{k+1} after a time step Δt_k

    // The time step t is chosen so that $J(S_{k+1}) \leq J(S_k)$



- Algorithm adapted from (Allaire et al. *J. Comp. Phys*. 2004)
- In-house programming in `Cast3m` (http://www-cast3m.cea.fr/)
- Redistancing performed using `mshdist` (Dapogny & Frey *Calcolo* 2012)
- Hamilton-Jacobi eq. solved using `advect` (Bui et al. *Int. J. Numer. Methods Fluids* 2011)

(A., Nika & Constantinescu CMAME 2022)

Numerical results

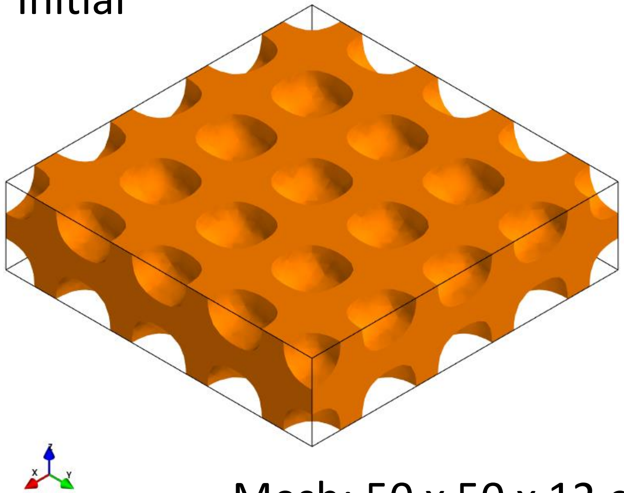
$$C^{\text{target}}$$

$$\left[\begin{array}{ccc|ccc} 0.12 & -0.06 & \star & \star & 2.3e^{-3} & \star \\ -0.06 & 0.12 & \star & 2.3e^{-3} & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \hline \star & 2.3e^{-3} & \star & 6.3e^{-4} & \star & \star \\ 2.3e^{-3} & \star & \star & \star & 6.3e^{-4} & \star \\ \star & \star & \star & \star & \star & \star \end{array}\right]$$

$$C^{*}$$

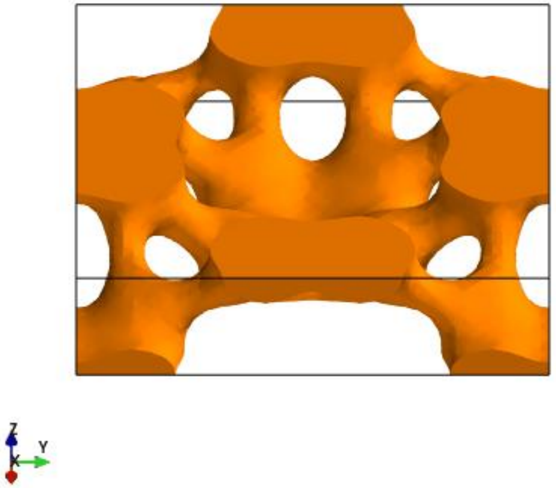
$$\left[\begin{array}{ccc|ccc} 0.097 & -0.033 & 0 & 2.9e^{-4} & 2.2e^{-4} & 0 \\ -0.033 & 0.098 & 0 & 2.7e^{-4} & 2.8e^{-4} & 0 \\ 0 & 0 & 0.023 & 0 & 0 & 1.8e^{-4} \\ \hline 2.9e^{-4} & 2.7e^{-4} & 0 & 2.7e^{-4} & 6.2e^{-5} & 0 \\ 2.2e^{-4} & 2.8e^{-4} & 0 & 6.2e^{-5} & 2.7e^{-4} & 0 \\ 0 & 0 & 1.8e^{-4} & 0 & 0 & 2.0e^{-4} \end{array}\right]$$

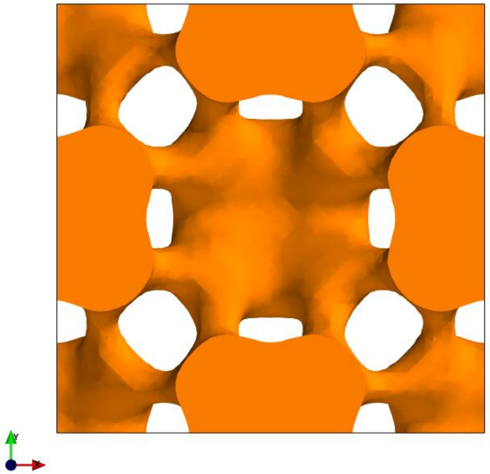
Initial



Mesh: 50 x 50 x 12 elts

Final

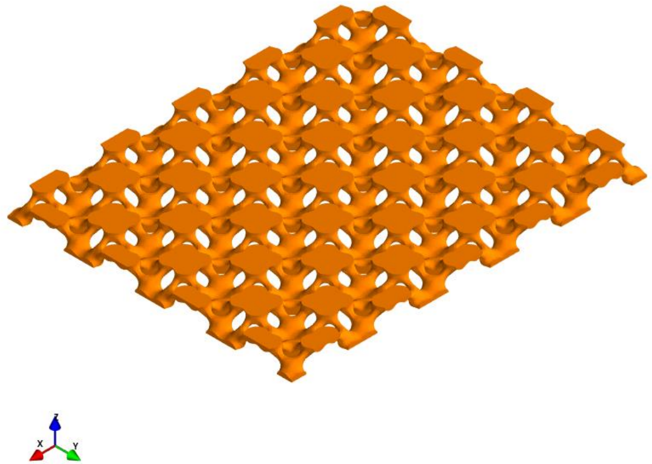


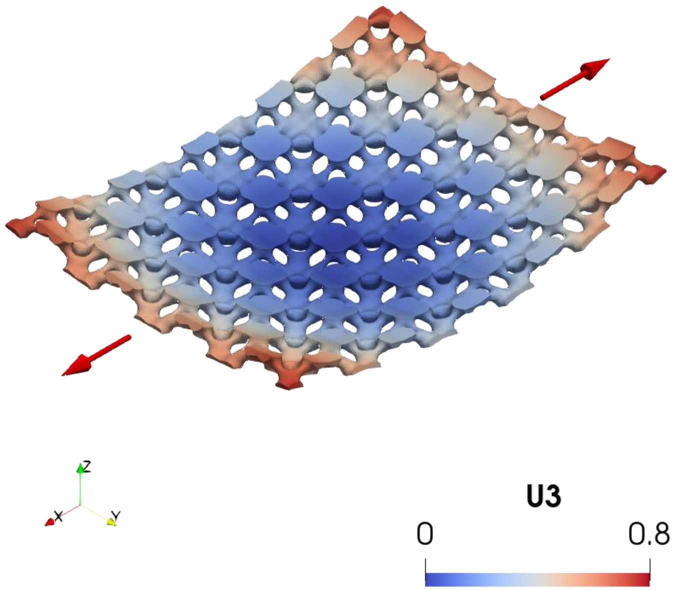


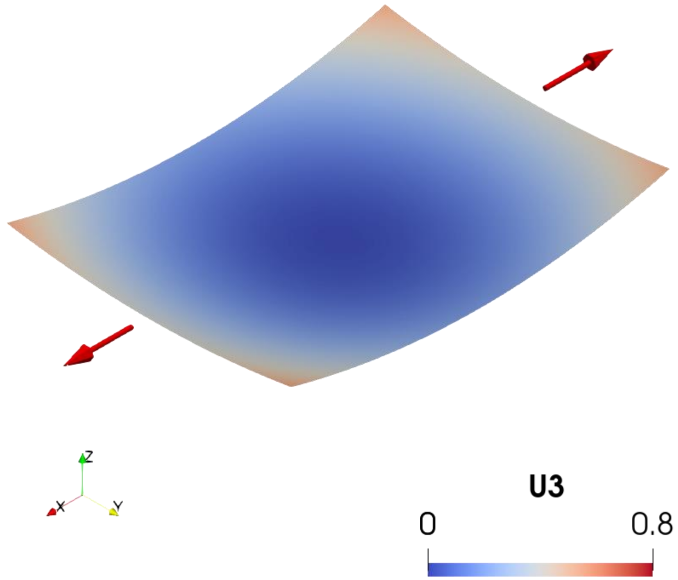
Response under uniaxial tensile test

$$S^* = \begin{bmatrix} 11.74 & 4.024 & 0. & -14.23 & -10.47 & 0. \\ 4.024 & 11.63 & 0. & -13.12 & -12.33 & 0. \\ 0. & 0. & 43.79 & 0. & 0. & -39.41 \\ -14.23 & -13.12 & 0. & 3934 & -878.1 & 0. \\ -10.47 & -12.33 & 0. & -878.1 & 3927 & 0. \\ 0. & 0. & -39.41 & 0. & 0. & 5035 \end{bmatrix}$$

Cast3M





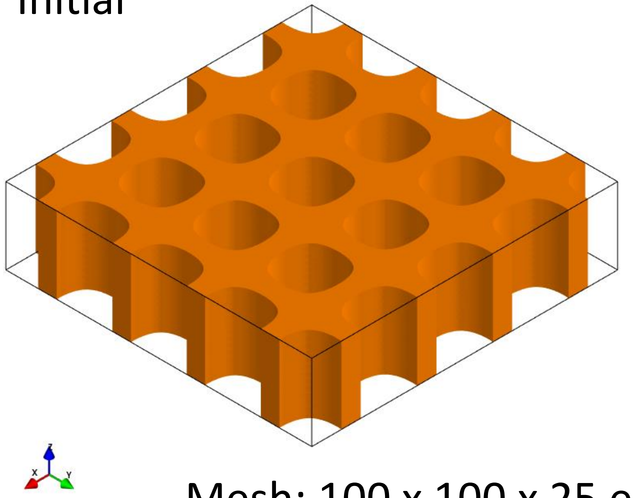


Numerical results

$$C^{\mathrm{target}} = \left[\begin{array}{ccc|ccc} 0.12 & -0.06 & \star & \star & 2.3e^{-3} & \star \\ -0.06 & 0.12 & \star & 2.3e^{-3} & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \hline \star & 2.3e^{-3} & \star & 6.3e^{-4} & \star & \star \\ 2.3e^{-3} & \star & \star & \star & 6.3e^{-4} & \star \\ \star & \star & \star & \star & \star & \star \end{array}\right]$$

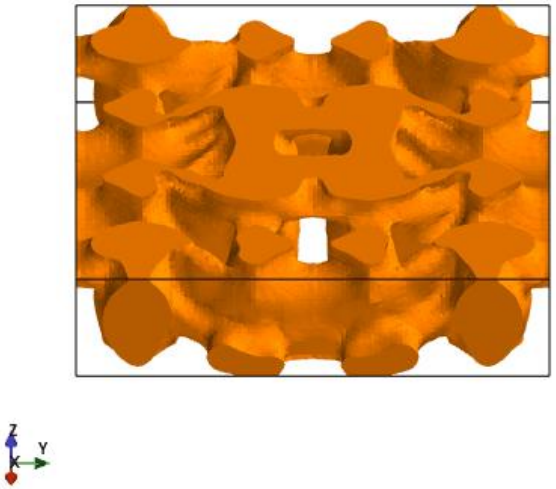
$$C^{*} = \left[\begin{array}{ccc|ccc} 0.120 & -0.059 & 0 & -1.5e^{-3} & 1.8e^{-3} & 0 \\ -0.059 & 0.119 & 0 & 1.7e^{-3} & 1.0e^{-4} & 0 \\ 0 & 0 & 0.03 & 0 & 0 & -2.7e^{-4} \\ \hline -1.5e^{-3} & 1.7e^{-3} & 0 & 6.1e^{-4} & -3.2e^{-5} & 0 \\ 1.8e^{-3} & 1.0e^{-4} & 0 & -3.2e^{-5} & 6.0e^{-4} & 0 \\ 0 & 0 & -2.7e^{-4} & 0 & 0 & 1.7e^{-4} \end{array}\right]$$

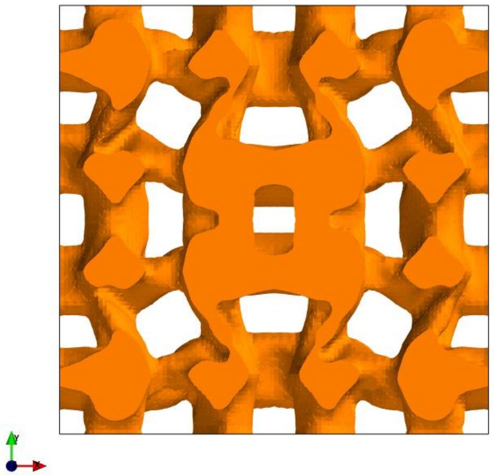
Initial



Mesh: 100 x 100 x 25 elts

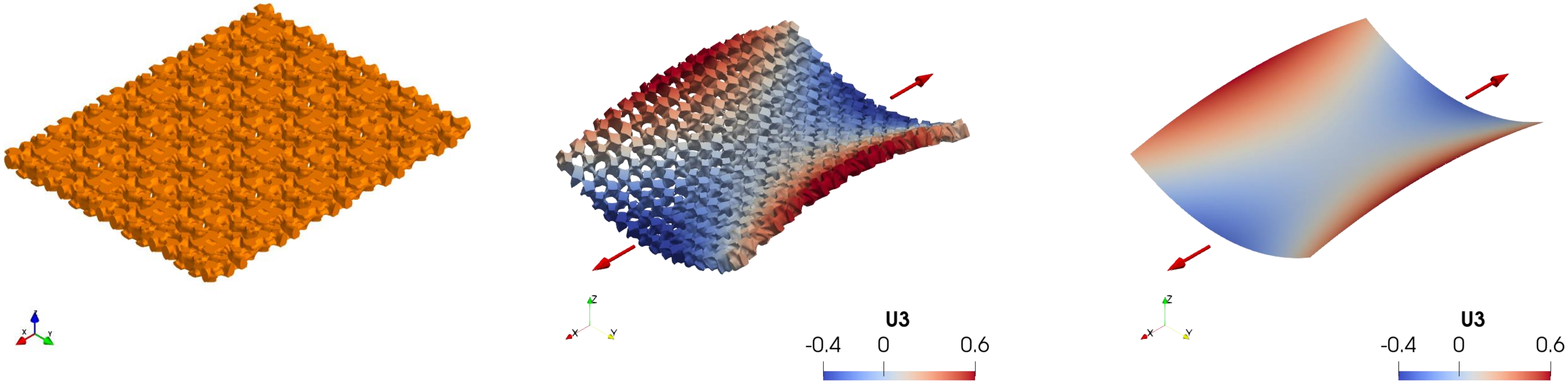
Final





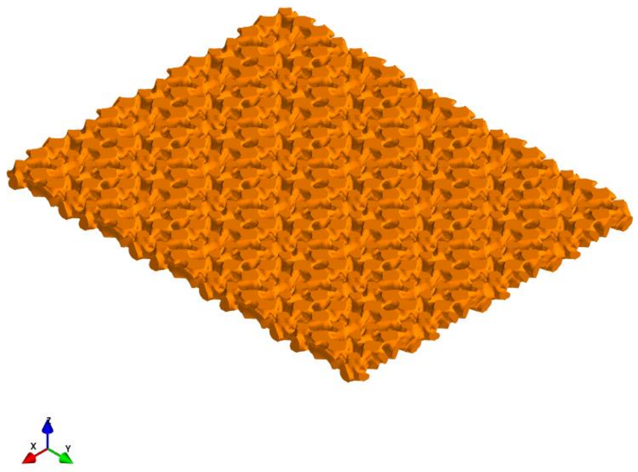
Response under uniaxial tensile test (direction X1)

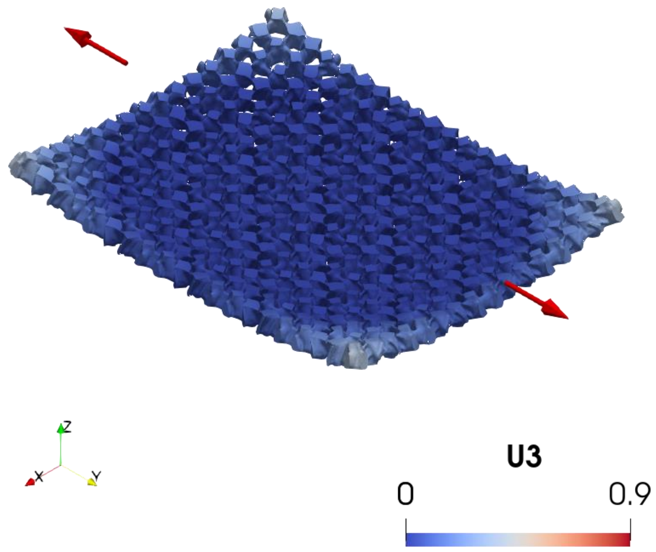
$$S^* = \begin{bmatrix} 11.83 & 5.736 & 0. & 11.23 & -35.85 & 0. \\ 5.736 & 11.54 & 0. & -19.10 & -20.15 & 0. \\ 0. & 0. & 33.82 & 0. & 0. & 53.71 \\ 11.23 & -19.10 & 0. & 1723 & 61.42 & 0. \\ -35.85 & -20.15 & 0. & 61.42 & 1781 & 0. \\ 0. & 0. & 53.71 & 0. & 0. & 5968 \end{bmatrix}$$

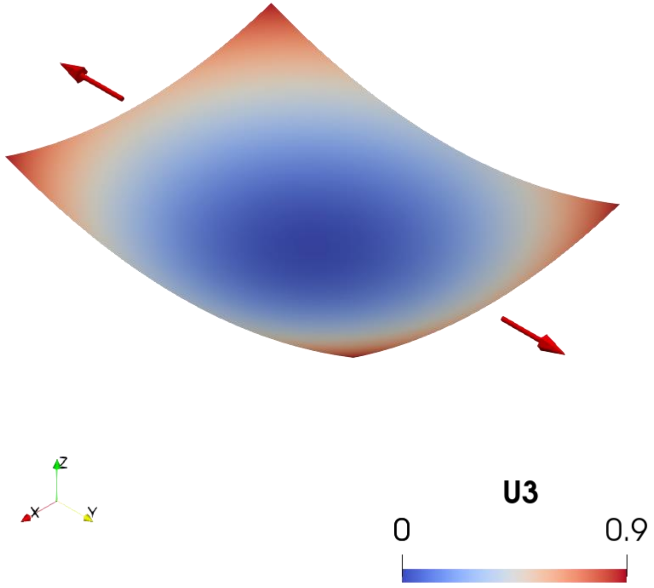


Response under uniaxial tensile test (direction X2)

$$
S^* = \begin{bmatrix}
11.83 & 5.736 & 0. & 11.23 & -35.85 & 0. \\
5.736 & 11.54 & 0. & -19.10 & -20.15 & 0. \\
0. & 0. & 33.82 & 0. & 0. & 53.71 \\
11.23 & -19.10 & 0. & 1723 & 61.42 & 0. \\
-35.85 & -20.15 & 0. & 61.42 & 1781 & 0. \\
0. & 0. & 53.71 & 0. & 0. & 5968
\end{bmatrix}
$$







Designed 3D lattices for thin panels: summary

Optimization of microstructure for shape-shifting panels

- Small strains
- Linear elasticity

Results

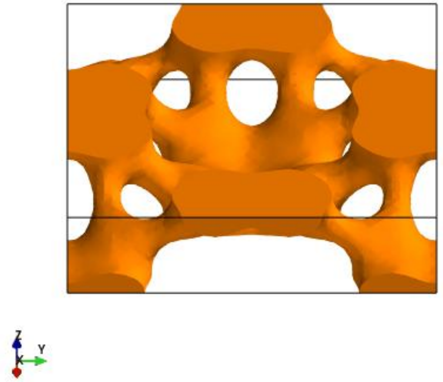
- Extensions of classical auxetic structures
- Novel organic shapes
- Good match of elastic moduli to targets

F. Agnelli, G. Nika, A. Constantinescu. “Design of thin micro-architectured panels with extension-bending coupling effects using topology optimization” *Comput. Meth. Appl. Mech. Eng.* 391 (2022)

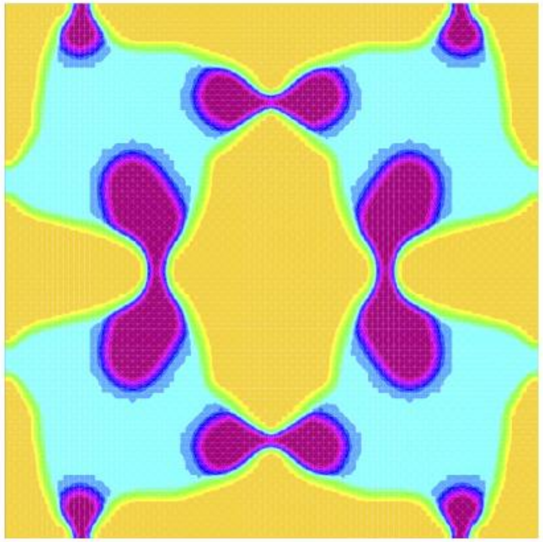
Perspectives & Future works

- Adapt the design methods to multimaterial framework
  - Wider possibilities for the design of panels

    (Agnelli et al. 2022, in preparation)



- Characterize & optimize multiphysics properties
  - Mechanical & Thermal properties
  - Mechanical & Magnetic properties



(Nika & Constantinescu *CMAME* 2018)

Thank you

filippo.Agnelli@mcgill.ca

https://filippoagnelli.wordpress.com