Karlstad Applied Analysis Seminar (2022)

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Compactness Property of the Linearized Boltzmann Operator for a Mixture of Polyatomic Gases

Abstract

In this talk we present a kinetic description for gas mixtures of polyatomic molecules. In particular, we will consider a model of the Boltzmann equation generalized from [5] (based on the procedure of Borgnakke and Larsen [3]) that describes a mixture of polyatomic gases of n species. At the microscopic level, we take into account the continuous microscopic internal energy I [6]. Under some convenient assumptions on the collision cross-sections we prove that the linearized Boltzmann operator L of this model is a Fredholm operator. For this, we write L as a perturbation of the collision frequency multiplication operator, and we prove that the perturbation operator K is a Hilbert-Schmidt integral operator using elementary arguments. We prove next that the collision frequency multiplication operator is coercive. These two results imply that L is a Fredholm operator. The compactness of K was indeed done by Grad [7] for a monoatomic single gas, and by [4] for a mixture of monoatomic gases, and by Bernhoff for polyatomic gases with discrete [1] or continuous [2] energy using a different approach. This is a joint work with Stephane Brull and Philippe Thieullen.

References:

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[3] C. Borgnakke, P.S. Larsen, Statistical collision model for Monte Carlo simulation of polyatomic gas mixture, Journal of Computational Physics, 18 (4): 405-420, 1975.

[4] L. Boudin, B. Grec, M. Pavic, and F. Salvarani, Diffusion asymptotics of a kinetic model for gaseous mixtures, Kinet. Relat. Models 6: 137-157, 2013.

[5] J.-F. Bourgat, L. Desvillettes, P. Le Tallec, and B. Perthame. Microreversible collisions for polyatomic gases and Boltzmann's theorem. European J. Mech. B Fluids, 13(2):237-254, 1994.

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[7] H. Grad Asymptotic theory of the Boltzmann equation, II, Rarefied Gas Dyn., Paris: 26-59, 1962.