# Prospective teachers designing tasks for dynamic geometry environments

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*The paper examines the quality of digitized tasks designed by 10 (small) groups of prospective upper secondary school teachers as part of a geometry course assignment. The results indicate that a small instructional intervention, addressing the planning and implementation of tasks in digitized task environments as well as how to stimulate students to make mathematical generalizations, led to a relatively high proportion (8 out of 10) of high-quality tasks designed by the prospective teachers.*

*Keywords: Task design, dynamic geometry environment, prospective mathematics teacher.*

## Background

Dynamic Geometry Environments (DGEs) have been used as educational tools for several decades (Sinclair et al., 2016). Mainly, it is the dragging function that is regarded as the defining feature of a DGE. By dragging points linked to geometrical objects, students can interact with these objects to search for regularities and invariances and to generate conjectures (e.g. Leung, 2011).

However, to utilize the potentials provided by DGEs, there is a need for carefully designed tasks. Indeed, designing DGE tasks or even evaluating existing tasks is not easy for teachers (Trocki & Hollebrands, 2018). To address this issue, researchers suggest models or principles for designing tasks that take advantage of DGEs as tools for exploration that might lead to conjectures, explanations and proofs (e.g. Fahlgren & Brunström, 2014; Leung, 2011). For example, Leung (2011) suggests a task design model composed of three epistemic modes that resemble different phases of the proving process: exploration, re-construction and explanation. These three modes are sequentially nested in the sense that one mode is a precursor for the next mode, which in turn, is a cognitive extension of the previous one. In this way, "…this task design model can be seen as a vehicle to carry the acquisition of mathematics knowledge." (Leung, 2011, p. 328).

In recent years, there has been an increasing interest in how to support teachers in their process of designing DGE tasks (e.g. Komatsu & Jones, 2019; Trocki & Hollebrands, 2018). While the study by Komatsu and Jones concerns specific task design principles to engage students in heuristic refutation, Trocki and Hollebrands provide a more generic framework with the intention to serve as guidance for teachers "…both for identifying and for writing high-quality tasks for DGSs [i.e. DGEs]." (p. 111). This framework, entitled the Dynamic Geometry Task Analysis Framework, is inspired by Smith and Stein's classification of tasks based on the level of cognitive demand that they require (Smith & Stein, 1998) as well as theories linked to various technological action linked to DGEs. Although Trocki and Hollebrands demonstrate the effect of the framework on teacher knowledge for recognizing and designing DGE tasks, they argue that this is only the beginning because there is a need for more research on investigating the usefulness of the framework (Trocki & Hollebrands, 2018).

Bozkurt and Koyunkaya (2020) address this request by investigating how prospective mathematics teachers (PMTs) developed their task design skills in DGE during a period of 14 weeks. Their study involved three cycles: (a) seminars on task design, followed by design of DGE tasks, (b) implementation of peer micro-teaching, followed by task revision, and (c) implementation in classrooms. Besides using Trocki and Hollebrands' (2018) framework as instructional material to develop the PMTs' skills in designing DGE tasks, the framework was used as a research tool to analyse task prompts as well as the questions posed and responses made by the PMTs during their teaching practices (Bozkurt & Koyunkaya, 2020). The micro-teaching cycles revealed that the PMTs were unable to reach neither the mathematical depth nor the technological actions that they planned for. However, Bozkurt and Koyunkaya found an improved development in PMTs' classroom practices after the micro-teaching. Having participated in each other's micro-teaching lessons, including follow-up discussions, the PMTs revised and developed their DGE tasks. Accordingly, Bozkurt and Koyunkaya suggest micro-teaching as an important component in teacher education courses aiming to develop PMTs' technology integration skills (2020). Moreover, they confirm the usefulness of the framework by Trocki and Hollebrands, both as instructional material and as a research tool.

In a similar study, Gulkilik (2020) examined DGE tasks designed by PMTs. The focus of this study was to examine in detail how PMTs' DGE tasks supported students' "…acquisition of mathematics knowledge"…(p. 2). To enable this, Gulkilik used Leung's (2011) model for task design. The PMTs were introduced to Leung's model and asked to analyse sample DGE tasks to examine their potential of engaging students in activities such as exploration, re-construction, and explanation, which relate to the three epistemic modes in Leung's model. In line with Bozkurt and Koyunkaya (2020), the PMTs implemented their designed tasks in micro-teaching with peers acting as students. To analyse the PMTs' tasks, Gulkilik developed a coding manual with descriptors related to each of the three epistemic modes as well as the transition between them, which enabled "…a continuous description of how PMTs guided students to mathematical understanding in DGE tasks." (2020, p. 13). One prominent finding was that the focus of the PMT tasks was on the construction of geometrical objects, i.e. without using pre-constructed sketches. Instead, the tasks included step by step instructions to build robust constructions, i.e. constructions were the properties are perceived under dragging. In this way, Gulkilik argues, the focus of the PMTs' tasks were limited to observe and explain invariants in the first constructed object, and "…did not utilize the potential of DGE to engage students in terms of exploration, re-construction, predicting, conjecturing, or proving…" (2020, p.13), i.e. activities for knowledge acquisition in DGE according to Leung's model (2011). Overall, the literature highlights the need for more research on task design within DGE to utilize the potential of the technology to reach a deeper mathematical understanding (Sinclair et al., 2016).

## Dynamic Geometry Task Analysis Framework

Trocki and Hollebrands' (2018) framework consists of two components: mathematical depth and technological affordances (see Table 1). Central in the framework are the prompts, i.e. questions or directions that require written (or oral) responses and/or technological actions. Besides the prompts, a DGE task most often includes a pre-constructed or partially constructed sketch of a geometrical object (Trocki & Hollebrands, 2018).

**Table 1: Dynamic Geometry Task Analysis Framework (Trocki & Hollebrands, 2018, p. 123)**

| **Allowance for Mathematical Depth** | | **Types of Technological Action** | |
|---|---|---|---|
| **Levels** | **Descriptions** | **Afford -ances** | **Descriptions** |
| N/A | Prompt requires a technology task with no focus on mathematics. | N/A | Prompt requires no drawing, construction, measurement, or manipulation of current sketch. |
| 0 | Prompt refers to a sketch that does not have mathematical fidelity. | A | Prompt requires drawing within current sketch. |
| 1 | Prompt requires student to recall a math fact, rule, formula, or definition. | B | Prompt requires measurement within current sketch. |
| 2 | Prompt requires student to report information from the sketch. The student is not expected to provide an explanation. | C | Prompt requires construction within current sketch. |
| 3 | Prompt requires student to consider the mathematical concepts, processes, or relationships in the current sketch. | D | Prompt requires dragging or use of other dynamic aspects of the sketch. |
| 4 | Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch. | E | Prompt requires a manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s). |
| 5 | Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships. | F | Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise (adapted from Sinclair (2003, p. 312). |

While the levels (0 to 5) of mathematical depth that a prompt allows reflect the progression of cognitive demand, the descriptors for technological actions (A to F) reflect the potential of using a DGE. It is the degree of coordination of mathematical depth and technological actions that indicate the quality (low, medium, high) of a DGE task (Trocki & Hollebrands, 2018).

Trocki (2015) reports findings from case studies involving three in-service teachers and three prospective teachers. Mainly, the research setup involved three parts: (a) design of a DGE task (with a given learning goal), (b) a tutorial session introducing the framework, and (c) redesign of the DGE task. By using the framework, Trocki found that all tasks redesigned by the participants increased their ranking to the highest level. Among the initial versions of the task, all (except for one) ranked medium in quality. Based on these findings, Trocki and Hollebrands suggest that the framework can serve as a useful tool in teacher education programs (Trocki & Hollebrands, 2018).

We had occasion to investigate this issue as part of a geometry course for prospective secondary and upper-secondary mathematics teachers. In contrast to the studies by Bozkurt and Koyunkaya (2020) and Gulkilik (2020), we only had a very limited amount of time at our disposal; one and a half week (over 6 weeks). Accordingly, we designed a small intervention, where the PMTs were asked to design DGE tasks as part of the course assignment. This paper aims to gain insight into what impact a small intervention might have on PMTs' abilities to design DGE tasks by exploring the following research question: *What quality of DGE tasks designed by PMTs can we expect of the instructional intervention?* As Bozkurt and Koyunkay's (2020) study, this paper is also guided by Trocki and Hollebrands' (2018) framework.

## Method

The study was conducted in spring 2020 in the context of a geometry course for PMTs in secondary and upper secondary school (ages 14–18) in Sweden. In total, 24 PMTs were enrolled in the course. Although the main aim of the course was to develop PMTs' content knowledge, in this case, both Classical Euclidean and Non-Euclidean geometries, there were some seminars on geometry teaching embedded in the course. Particularly, these seminars intended to develop PMTs' skills in planning and implementation of tasks in digitized task environments (such as DGE), as well as their abilities to stimulate each students' learning in the ordinary classroom, including those students who easily reach the knowledge requirements. The instructional intervention addressed both these issues by offering a seminar that introduced geometrical tasks whose numerical solutions could be developed into general results on giftedness and one homework with a follow-up seminar on task design in DGE. After the intervention, as part of the course assignment, the PMTs were asked to design (in pairs or small groups) DGE tasks for (upper) secondary school for all students at different levels of knowledge. Preliminary versions of the tasks were trialled by peers, who provided both oral (at a small seminar) and written responses. The PMTs were then expected to revise and provide a final version of the DGE tasks. Although the participating PMTs were familiar with dynamic mathematics environments as learners of mathematics, the role as task designers were new to them.

### The intervention

The mentioned seminar, based on a systematic review (Szabo, 2017) was performed by the second author of this paper, and highlighted the importance of designing tasks that offer opportunities for students to reach general solutions, thereby addressing students performing at higher qualitative levels. To achieve this, we suggested using DGE tasks, in which the participants were encouraged to explore mathematical relationships, to make and verify conjectures, to generalize (if possible) and eventually to construct a proof (Fahlgren & Brunström, 2014). To introduce the ideas behind Trocki and Hollebrands' (2018) framework, the PMTs were encouraged to perform a homework as a preparation for a follow-up seminar. As a basis for the homework, we used three versions of a sample task, provided by Trocki and Hollebrands, to demonstrate various levels of DGE task qualities. These tasks address "…the same two learning goals: 1) justify that opposite angles of parallelograms are congruent; 2) justify that the diagonals of parallelograms bisect each other." (p.127). Each of the tasks consists of a combination of a sketch of a parallelogram and some associated prompts for students to achieve the learning goals. The homework included a brief introduction to the task, of which three versions, A, B and C were provided, and the two learning goals (as described above), followed by some prompts (see Figure 1). At the follow-up seminar, the PMTs discussed the homework in small groups before a whole-class discussion. The focus of these discussions was on to what extent the three versions of the task took advantage of the DGE. For example, which of the versions (A, B, and C), if any, encourage students to explore and discover mathematical relationships.

| | | |
|---|---|---|
| (a) | Start by constructing a parallelogram in GeoGebra. Make sure that your construction is robust, i.e. that the properties of the parallelogram are perceived even when one of its vertices is dragged. |
| (b) | Perform the three versions (A, B and C) of the task. Reflect on possible constraints and opportunities that each version entails for a student to achieve the learning objectives. |
| (c) | Reflect on the quality of the different versions of the tasks by considering the following questions:<br>• How is the potential of the DGE utilized?<br>• What is it that makes one task of higher quality than another? |
| (d) | How can the task be adapted for students who easily reaches the knowledge requirements? Give suggestions. |

**Figure 1: The homework prompts**

## Data collection and analysis

The unit of analysis was the DGE tasks (both the preliminary and the final version) designed by 10 groups (A to J) of PMTs, and the written responses from peers. Each task included a number of prompts for potential students. Some tasks also included pre-constructed (manipulable) sketches. For each task, all prompts were coded with Trocki and Hollebrands' (2018) framework. The coding process was done independently by two of the authors of this paper and then comparisons were made followed by discussions (between all authors) until full agreement was reached. Although the framework was straightforward to use, some subtleties emerged, which are also recognized by Trocki and Hollebrands (2018). First, the distinction between the mathematical depth codes 4 and 5. According to Trocki and Hollebrands, the

> [c]hoice of the word *explain*, as opposed to *justify* or *prove*, was deliberate, in that it serves expose the student to the need *for* explanation as opposed to a particular type *of* explanation (e.g. deductive proof). The code is also based on research that emphasizes a need for students to explain what they notice when using a DGS. (p. 124)

Another subtlety concerns the technological action codes E and F. To sort this out, we needed reexamine Trocki´s original work (2015). A prompt is considered a code E when it "…requires manipulation and directs the student on what to notice." (p.173), while a code F was used if the manipulation is based on a student conjecture, i.e. "…not on a preconceived conclusion on behalf of the task writer."(p.174) .

When all prompts related to a task have been coded, they are assessed holistically to define the quality of the task ackording to the three levels described by Trocki and Hollebrands:

> **Low:** The task *does not* contain a collection of prompts that co-ordinate mathematical depth and technological actions in such a way as to *require* the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch.
>
> **Medium:** The task contains a collection of prompts that co-ordinate mathematical depth and technological actions in such a way that *may encourage but does not necessitate* that the student make generalized conclusions based on emergent invariant relationships that go beyond a static sketch. (2018, p.126)
>
> **High:** The task contains a collection of prompts that co-ordinate mathematical depth and technological actions in such a way that *requires* the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch. (2018, p.125)

For each of the 10 DGE tasks, all associated prompts were coded by indicating the level(s) of mathematical depth and the type(s) of technological action (see Table 1). This coding generated 10 individual summary tables, which formed the basis for ranking the task quality. To illustrate the coding process, we use one of the tasks, designed by Group D (see Figure 2).

| **Prompt** | **Code (within brackets) and Explanation** |
|---|---|
| 1. Create an arbitrary quadrilateral (convex) with the tool "Polygon". Remove the labels on the sides of the quadrilateral. Then mark the midpoints on each side of the arbitrary quadrilateral. Use the "Polygon" tool to construct the inscribed quadrilateral. | **(1,A,C)**<br>To "create an arbitrary quadrilateral" (coded A) and to "mark the midpoints" (coded C) in Prompt 1, the students need to recall the definition of a (convex) quadrilateral (coded 1). |
| 2. Formulate a hypothesis for the type of geometric figure that is created when the midpoints are connected. Write down your hypothesis on paper. Also, try to drag the corners of the original quadrilateral, before formulating your hypothesis. | **(2,3,D)**<br>Students are asked to drag (coded D) the figure created to formulate a hypothesis (coded 3) about the type of geometric figure it represents (coded 2). |
| 3.After formulating your hypothesis, read the length of the sides and measure the angles of the inscribed quadrilateral. Also, try to drag the corners of the original quadrilateral to see any relationships. Does this result agree with your hypothesis? What type of geometric figure did you get and what characterizes one? If your result is incorrect, justify why and state what assumptions you made that were incorrect and what should have been your correct conclusion. Write down all conclusions on paper. | **(3,4,5,B,D,E,F)**<br>The codes of technological action emerged due to the requirement of measuring the angles of the inscribed quadrilateral (coded B), and then to drag (coded D) the corners to obtain multiple examples (coded E) from which one can generalize to "…see any relationships" (coded F). Concerning the mathematical depth, the students are encouraged to consider relationships in the current sketch (coded 3), and to justify (coded 4) the hypothesis from Prompt 2. Since the prompt requires the student to go beyond the current construction and generalize the mathematical relationships, it receives a code 5. |
| 4. Formulate a hypothesis about the relationship between the area of the inscribed quadrilateral and the area of the original quadrilateral. Write down your hypothesis on paper. Also, try to drag the corners of the original quadrilateral to try to see connections before formulating the hypothesis. | **(2,3,D)**<br>Coded in the same way as Prompt 2. |
| 5. Measure the area of the inscribed quadrilateral and the original quadrilateral. Drag the corners of the original quadrilateral to discover interesting relationships. Does the result agree with the hypothesis that you formulated in point 5? Write down your conclusions on paper. | **(3,5,B,D,E,F)**<br>Coded in the same way as Prompt 3, except for code 4. In contrast to Prompt 3, Prompt 5 does not require the student to justify the conclusion (code 4). |

**Figure 2: Analysis of one of the DGE tasks (Group D) designed by the PMTs**

Since the task includes prompts that co-ordinate mathematical and technological actions in ways that requires students to draw generalized conclusions (code 5) based on emergent invariant relationships that go beyond a static sketch, we ranked the quality of the task as 'high'. During the quality ranking process, we compared and contrasted our interpretations with those made by Trocki (2015).

## Results and discussion

Table 2 shows our task ranking of the DGE tasks designed by the 10 groups (A-J). Notably, there was no difference in terms of task ranking between the preliminary and final versions of the DGE tasks. The reason for this might be that the written feedback provided by peers foremost concerned clarification of the DGE tool instructions and/or formulations of questions. We also (in Table 2) indicate whether the tasks provide students with manipulable pre-constructed sketches or step-by-step guidance for constructing geometrical figures.

**Table 2: Overview of the results**

| Group | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| **Task ranknint** | High | High | High | High | High | High | Medium | High | High | Medium |
| **Pre-constructed sketch?** | No | Yes | No | No | Yes | Yes | No | No | No | No |

As seen in Table 2, 8 (out of 10) tasks ranked high, which indicates that the instructional intervention worked well. However, these results should be interpreted with caution. Besides the subtleties concerning the coding of prompts indicated by Trocki and Hollebrands (2018), the subjective nature of the task ranking method must be taken into consideration. In contrast to Trocki's (2015) study, the tasks designed in this study aimed to address different learning goals, which made the holistic analyses of the prompts associated with a specific task challenging due to fewer comparison opportunities between the tasks. So, for example, to distinguish between 'may encourage but does not necessitate' (medium') and 'requires' (high), was not straightforward. Consequently, we argue that this perspective may affect the validity of our study. Therefore, the relatively high proportion of high-quality tasks designed by the PMTs can be questioned. Still, the tasks ranked as high quality according to the definition (Trocki & Hollebrands, 2018), indeed include prompts that coordinate mathematical depth and technological actions. For example, in this study, the tasks ranked 'high' all offered opportunities for students to reach a generalization beyond the DGE sketch (Code 5) by directions for technological actions such as dragging to recognize invariants in the sketch (Code D and E). A possible explanation for the relatively high proportion of tasks including generalization-making prompts might be the first seminar in the intervention. This seminar highlighted, among other things, the importance of encouraging mathematical generalizations as a way to challenge high-achieving students. Nevertheless, reconsidering the comparatively small size of the instructional intervention, we argue that it was successful in that most of the PMTs designed DGE tasks were ranked at high quality, at least according to Trocki and Hollebrands' (2018) framework.

Moreover, Table 2 shows that several groups did not provide pre-constructed sketches in their tasks. This is in accordance with Gulkilik's (2020) finding that PMTs' tasks provided students instructions to make (robust) constructions on their own. There are several possible explanations for this result. In previous courses, the participating PMTs experienced, as learners, tasks designed for dynamic environments (although not DGE) that offer construction guidance rather than providing pre-constructed sketches. Further, the homework (see Figure 1) prompted the PMTs to make a robust construction before examining the sample tasks, which might have influenced their task design. Moreover, in contrast to previous studies utilizing Trocki and Hollebrands' framework (e.g. Bozkurt & Koyunkaya, 2020; Trocki, 2015), the PMTs in this study were not introduced to the framework itself. Instead, they were asked to examine the quality of three sample tasks, as the PMTs in Gulkilik's (2020) study. Since Trocki and Hollebrands' framework is strongly influenced by the work of Sinclair (2003), who provides guidance for designing tasks utilizing pre-constructed DGE sketches, the introduction of the framework to the PMTs might affect their choice of providing pre-constructed sketches, which was the case in Bozkurt and Koyunkaya's (2020) study.

To sum up, despite its limitations (e.g. no data was collected during the intervention), this study adds some different perspectives to the emergent research field of DGE task design (Sinclair et al., 2016), particularly in teacher education programs (Trocki & Hollebrands, 2018). This study confirms the usefulness of Trocki and Hollebrands' (2018) framework as instructional material, although not necessarily by presenting the framework itself but by asking teachers to evaluate the quality of sample tasks. As a suggestion for further research, we propose deepening this study by analysing all steps of the intervention, not only its outcome (in this case the designed DGE task and associated written responses from PMTs). We also suggest comparing the usefulness of this framework with the suggested operationalization of Leung's model by Gulkilik (2020) to analyse the educational potential provided by DGE tasks.

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