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Optimization on manifolds: methods and applications – an engineering perspective

Abstract

Numerical optimization often incorporates constraints. These constraints can be satisfied by using, for example, the penalty method or introducing Lagrange multipliers. These methods often introduce unsatisfactory features to the optimization process, e.g., the dependence on a penalty parameter or the transformation of a minimization problem into a saddle-point problem. Luckily, such constraints can sometimes be interpreted as restrictions to the design space. As a naïve example, if the optimization problem is to find a point P in R2 with the objective of minimizing the distance to a given point A and the constraint that the distance to a given point B should have a distinct value r, this is equivalent to the unconstrained optimization problem of finding a point P on a circle around B with radius r that minimizes the distance to A. In mathematical terms, this results in the design space being a nonlinear manifold. Therefore, it proves to be beneficial in many cases to think about constrained optimization problems in terms of differential geometry.

It allows the transformation of a problem from "constrained optimization on an unconstrained space" to "unconstrained optimization on a constrained space", which retains the structure of the problem and allows a dimensional reduction of the design space. Unlike penalty methods, it exactly fulfills the constraint and does not require user defined parameters. In contrast to the Lagrangian multiplier method, the minimization problem is not changed

to a saddle point problem. The transformation to an unconstrained optimization problem on a constrained space requires careful generalization of innocent-appearing concepts. For example, it includes the generalization of the incremental update of design variables. If they live on a manifold (instead of a linear vector space), simple addition of the incremental update and the design variable is not well-defined and alternative concepts are needed, like addition in tangent spaces and exponential mapping. Similar consequences apply to other components of an optimization toolchain for the nonlinear manifold setting. This talk exemplarily presents concepts of differential geometry for geometric optimization algorithms from an engineering perspective. Additionally, applications in simulation of physical processes, e.g., elastic deformation of shells and micromagnetostatics, are presented.