Karlstad Autumn PhD School
Interacting Particles meet Homogenization and Measure Theory

Micol Amar (Rome, IT)
Hong Duong (Birmingham, UK)
Sander Hille (Leiden, NL) – lecturers
Adrian Muntean (Karlstad, SWE) – organizer

November 24, 2021

Aim of the school. Target audience

The expected audience to this online autumn school on *Interacting Particles meet Homogenization and Measure Theory* (**Nov. 29, 30, Dec. 1st, 2021**) are PhD students, advanced master students, and researchers working in the field of applied analysis with interest in multiscale modeling, mathematical analysis, and numerical simulation of stochastically interacting particles and/or partial differential equations.

The event includes three mini-courses. The chosen topics and working techniques are meant to prepare the researcher to work on complex mathematical models where either microstructures, stochastic fluctuations, defects, or loosely-controlled perturbations interact via multiple space scales with evolution equations. Situations of this type arise in material and life sciences, and one of our roles as mathematicians is to provide suitable reduction techniques where the trust into the "model reduction" procedure can be guaranteed in terms of either rigorous or computable estimates.

It is advisable that the true beginner looks inside some of the indicated reading materials (e.g. Refs. [3] and [2] in Amar's course, Refs. [1] and [2] in Duong's course and Ref. [1] and [3] in Hille 's course.)

Each mini-course will propose take-home assignments.

Course content

Each mini-course includes three slots of cca. 45 minutes. The breaks between lectures will offer opportunities for online networking. The precise course content, a few bibliographic hints, as well as a short bio of each of the lecturers are indicated in the following.

Schedule

The event will take place via zoom. The zoom links for the mini-courses as well as of the networking moments will be announced later on, after the end of September.
Here is the schedule of the lectures (Swedish time):

Day 1 (Nov. 29)
Muntean (8h25-8h30) opening
Amar 1 (8h30-9h15)
Hille 1 (9h30-10h15)
Amar 2 (10h30-11h15)

Day 2 (Nov. 30)
Hille 2 (8h30-9h15)
Amar 3 (9h30-10h15)
Duong 1 (10h30-11h15)
Pitch talks (11h30-12h06)

Day 3 (Dec. 1st)
Hille 3 (8h30-9h15)
Duong 2 (9h30-10h15)
Duong 3 (10h30-11h15)
Muntean (11h15-11h20) closing

Pitch talks

On the last part of Day 2, we have the following 5 pitch talks (*6 min./pitch, max. 3 slides per speaker*):

Name: **Apratim Bhattacharya**, *FAU Erlangen-Nürnberg, Germany*

Title: *Homogenization of a nonlinear drift-diffusion system for multiple charged species in a porous medium*

Name: **Patrick van Meurs**, *Kanazawa University, Japan*

Title: *Mean field limits of deterministic interacting particle systems*

Name: **Chiharu Kosugi**, *Japan Women's University, Tokyo, Japan*

Title: *Existence and uniqueness of solutions for the initial and boundary value problems representing motions of the elastic materials*

Name: **Surendra Nepal**, *Karlstad University, Sweden*

Title: *Error estimates for semi-discrete finite element approximations for a moving boundary problem capturing the penetration of diffusants into rubber*

Name: **Vishnu Raveendran**, *Karlstad University, Sweden*

Title: *Scaling effects on the periodic homogenization of a reaction-diffusion-convection problem posed in homogeneous domains connected by a thin composite layer*

Name: **Nacira Agram**, *Linnaeus University, Sweden*

Title: *Fokker-Planck PIDE for McKean-Vlasov jump diffusions with application to HJB equation.*

Zoom link for the event

If you have not received it yet the zoom link in your email box, then contact A. Muntean.

Certificate of attendance

A participation certificate as well as 3 ECTS can be awarded, should this be requested. For this matter, contact Adrian Muntean, prof. dr. habil. (email: adrian.muntean@kau.se).

Acknowledgments

The organizer thanks the Swedish Research Council (grant nr. VR2018-03648, project Homogenization and Dimension Reduction of Thin Heterogeneous Layers*) as well as the Magnussonsfond of KvA (grant nr. MG2020-0008,* Measure-valued Multiscale Evolution Equations: Analysis and Approximation*) for financially supporting his research and this autumn school. Additional financial support for this event has been provided by Karlstad University as well as via the action MiMM Day® (*Mathematics with Industry Day*).*

Mini-course on:

Homogenization results for Fick and Focker-Plank equations in composite media with periodic microstructures

Micol Amar[1]

Key-words: *Homogenization; oscillating capacity; porous media; asymptotic expansion; multi-scale analysis.*

- **Lecture 1:**

  Brief motivation of the physical problem.

  Introduction to the formal expansions in the periodic homogenization.

  Example: formal homogenization of the classical parabolic equation.

  Reference: [3]

- **Lecture 2:**

  Formal homogenization of the Fick equation with oscillating capacity in the two regimes: fast oscillations and slow oscillations.

  Reference: [1]

- **Lecture 3:**

  Formal homogenization of the Focker-Planck equation with oscillating capacity, in the two regimes: fast oscillations and slow oscillations.

  Reference: [2]

Further references on the unfolding technique, for those who are interested in the rigorous proofs of the presented results: [4, 5, 6].

References

[1] *M. Amar, D. Andreucci, and D. Bellaveglia. The time-periodic unfolding operator and applications to parabolic homogenization.* Atti Accad. Naz. Lincei Rend. Lincei Mat. Appl.*, 28:663–700, 2017.*

[2] *M. Amar, D. Andreucci, and E.N.M. Cirillo. Diffusion in inhomogeneous media with periodic microstructures.* Zeitschrift für Angewandte Mathematik und Mechanik*, 2021, online version DOI: 10.1002/zamm.202000070.*

[3] *A. Bensoussan, J. L. Lions, and G. Papanicolaou.* Asymptotic Analysis for Periodic Structures. *North Holland, Amsterdam, 1978.*

[1]Dipartimento di Scienze di Base e Applicate per l'Ingegneria, Sapienza - Università di Roma, Via A. Scarpa 16, 00161 Roma, Italy.
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[4] *D. Cioranescu, A. Damlamian, and G. Griso. Periodic unfolding and homogenization.* Comptes Rendus Mathématique*, 335(1):99–104, 2002.*

[5] *D. Cioranescu, A. Damlamian, and G. Griso. The periodic unfolding method in homogenization.* SIAM J. Math. Anal.*, 40(4):1585–1620, 2008.*

[6] *D. Cioranescu, A. Damlamian, and G. Griso.* The periodic unfolding method. Theory and Applications to Partial Differential Problems*, volume 3 of* Series in Contemporary Mathematics*. Springer, Singapore, 2018.*

Short bio

Micol Amar *received her degree in Physics (*cum laude*) from the University of Milan (Italy) in 1988. In 1990, she obtained her Magister Philosophiae (*cum laude*) and in 1993 her PhD in Functional Analysis from S.I.S.S.A. - I.S.A.S. (International School of Advances Studies) in Trieste (Italy). In 1992 and 1996 she gets a CNR fellowship for a research period in Paris (France) at Université Pierre et Marie Curie (Paris VI). From October 1992 to October 1998 she is researcher at the University of Pavia and from November 1998 up today she is Associate Professor at the University "La Sapienza" of Rome (Italy).*

In 2013 and again in 2018 she obtained the so-called Italian ASN for full professor in Mathematical Analysis. Her research interests mainly concern Calculus of Variations and Partial Differential Equations, with a particular attention to periodic homogenization problems arising in modeling physical properties of composite media.

Mini-course on:

Collective dynamics of interacting particle systems

Hong Duong[2]

Key-words: *Stochastic dynamics; McKean-Vlasov system; coupling method; mean-field limits; approximation schemes; model reduction.*

Complex systems in natural and applied sciences (such as molecular systems, crowd dynamics, swarming, opinion formation, etc) are often described by large systems of interacting particles (agents). These interacting particle models provide a detailed description of the underlying phenomena at a microscopic/mesoscopic level. However, in most practical applications, it is often unnecessary and impossible to deal with the dynamics of each individual particle due to the curse of dimensionality. Deriving and understanding the collective dynamics (emerging phenomena, macroscopic behaviour) from the (microscopic) interacting particle systems is a major challenge due to their inherent complexity, but is of crucial importance. The macroscopic (coarse-grained) models, which are often nonlinear nonlocal partial differential equations, reduce the degree of freedom focusing only on quantities of interest and thus are often more computational tractable using existing numerical methods.

The aim of this series of lectures is to present important methods and techniques for a rigorous derivation of the collective dynamics of stochastic interacting particle systems, as well as an analysis of the coarse-grained models. The following topics will be presented: the coupling method, mean-field limits and the long-time behaviour of McKean-Vlasov dynamics (lecture 1), variational approximation schemes for non-gradient systems (lecture 2), and model reduction of McKean-Vlasov dynamics (lecture 3). The lectures are based on the papers [1, 2, 4, 3, 5, 6]. If time permits, some recent developments in the fields [7, 8] will also be discussed.

References

[1] *F. Golse, On the Dynamics of Large Particle Systems in the Mean Field Limit.* Lecture Notes in Applied Mathematics and Mechanics*, Eds. A. Muntean, D. Rademacher, A. Zagaris, vol. 3, Springer, 2016.*

[2] *M. H. Duong. Long time behaviour and particle approximation of a generalised Vlasov dynamic,* Nonlinear Analysis: Theory, Methods & Applications*, Volume 127, Pages 1-16, 2015.*

[3] *M. H. Duong, A. Lamacz, M. A. Peletier, A. Schlichting, and U. Sharma. Quantification of coarse-graining error in Langevin and overdamped Langevin dynamics.* Nonlinearity*, 31(10), pp. 4517-4566, 2018.*

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[2]School of Mathematics, University of Birmingham, UK.
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[5] *M. H. Duong and G. A. Pavliotis. Mean field limits for non-Markovian interacting particles: convergence to equilibrium, GENERIC formalism, asymptotic limits and phase transitions.* Comm. Maths. Sci.*, Volume 16, No.8, page 2199-2230, 2018.*

[6] *D. Adams, M. H. Duong and G. dos Reis. Entropic regularisation of non-gradient systems.* `arXiv:2104.04372`*.*

[7] *P. E. Jabin, Z. Wang, Quantitative estimates of propagation of chaos for stochastic systems with $W^{-1,\infty}$ kernels.* Invent. math. *214, 523–591, 2018.*

[8] *S. Serfaty (appendix with M. Duerinckx) Mean Field Limit for Coulomb-Type Flows.* Duke Math. J. *169, No. 15, 2887-2935, 2020.*

Short bio

Hong Duong *received his PhD in Mathematics (*cum laude*) in September 2014 from the ITN Marie-Curie PhD Fellow Programme jointly between University of Bath (UK) and Eindhoven University of Technology (the Netherlands). From October 2014 to September 2017, he was a Research Fellow at Mathematics Institute, University of Warwick (UK). From October 2017 to July 2018, he held a prestigious Chapman Fellow at Department of Mathematics, Imperial College London (UK).*

Since July 2018, he became an Assistant Professor in School of Mathematics, University of Birmingham (UK). Hong Duong's research interests span a wide range of topics in the intersections of analysis and applied probability, including partial differential equations, interacting particle systems, non-equilibrium thermodynamics, and evolutionary game theory. Most of his research are inspired from applications in statistical physics and biological/social/material sciences. His research has been supported from the ITN "Fronts and Interfaces in Science and Technology" (EU), the NWO (Netherlands), the London Mathematical Society (UK) and the EPSRC (UK).

Mini-course on:

A measure theoretical framework: Linking continuum and individual-based models

Sander Hille[3]

Key-words: *Measure-valued evolution equations; semigroups on measures; approximation techniques; modeling with measures; discrete-to-continuum coupling.*

In the mathematical modelling of natural phenomena the modeller has to weigh detail versus the particular aim of the modelling effort. Detailing is the inclusion of smaller scale processes in the mathematical description, for example based on deeper scientific understanding of these underlying processes. In individual-based models one identifies 'individuals' (e.g. molecules, material defects, organisms, etc.) and prescribes how they interact with each other. This leads to a high dimensional system of coupled equations; each equation describes the detailed behaviour of an individual: often called interacting particle systems. If characteristics of interest of classes of individuals are approximately the same, one may derive population-level approximations of the dynamics: suitable continuum models for the behaviour of the population density function(s) in time.

These densities are considered with respect to a reference measure (a priori fixed): usually Lebesgue measure. In these lectures we present a measure theoretical framework for considering both a continuum and an individual-based description within this single framework. This provides analytical advantages. Continuum approximations in a function space setting may suffer from blow-up, eg. when aggregation takes place of individuals on a lower dimensional part of state space. Using a measure framework can overcome such issues. Moreover, after introducing functional analytic and metric topologies on spaces of measures, one can consider the question of quantifying the quality of approximation of one description by the other within this framework too.

In the lectures we shall introduce, explain and provide an overview of fundamental results on relevant metrics and topologies on spaces of measures in view of this question, providing also the latest insights [1, 2]. By means of examples (e.g. [3, 4, 5]), we show how dynamics in such spaces of measures can be formulated mathematically and subsequently analysed. To that end, we also introduce the concept of (Bochner) integration in these spaces of measures and its elementary properties.

After following these lectures the participant has an overview over the relevant concepts and results of this measure framework, a good set of entrance points to the relevant literature and main researchers in the field and initial ideas and inspiration for starting application of this framework in the context of mathematical questions of his/her interest.

References

[1] S.C. Hille, and D.H.T. Worm (2009). Embedding of semigroups of Lipschitz maps into positive linear semigroups on ordered Banach spaces generated by measures. Integr. Equ. Oper. Theory

[3]Mathematical Institute, University of Leiden, The Netherlands.
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[2] S.C. Hille, T. Szarek, D.T.H. Worm, and M.A. Ziemlańska (2021). Equivalence of equicontinuity concepts for Markov operators derived from a Schur-like property for spaces of measures, Stat. Prob. Letters **169***: 108964.*

[3] J. Evers, S.C. Hille, and A. Muntean (2015). Mild solutions to a measure-valued mass evolution problem with flux boundary conditions. J. Diff. Equ. **259***: 1068–1097.*

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Short bio

Sander Hille *received his PhD in Mathematics in June 1999 from Leiden University, the Netherlands. He was then researcher and interdisciplinary research project manager at the Telematica Institute, Enschede till May 2003. He then returned to Leiden University as assistant professor to set-up a research line and educational track on the topic of Mathematical Biology. This resulted (among others) in a multidisciplinary research collaboration on the topic of auxin transport in plant biology, merging experimental biological research and mathematical analysis. This mixed mathematical-experimental research is a much inspiring and fruitful effort to this day, with various projects within the associated Plant BioDynamics Lab (PBDL). Some concern multiscale models and data-driven analysis of these models.*

On the more fundamental mathematical side, Dr. Hille is interested in mixed stochastic-deterministic models (related to biological phenomena): so-called Piecewise Deterministic Markov Processes (PDMPs). His research interests may be summarized under the general abstract mathematical theme 'Dynamical Systems in Spaces of Measures', which includes the functional analytic and topological properties of spaces of finite signed Borel measures discussed in the lectures.

Dr. Hille (co-)organized and participated in various workshops on the topic of 'Modelling with Measures', on global and local optimization and (PhD-)courses on advanced measure theory and the (functional) analysis in spaces of measures.