Task Design with Focus on Exploration, Explanation and Generalization using GeoGebra

The Seventh Nordic GeoGebra Conference

Trondheim, September 25th, 2016

Mats Brunström and Maria Fahlgren

The original proof task

Let P be an arbitrary point on the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
and let F(ae,0) be a focus.
Let M be the midpoint of FP.
Prove that the locus of M is an ellipse.

A mathematical situation – a sequence of tasks

Let *P* be an arbitrary point on an ellipse. Let *M* be the midpoint between P and one of the foci.

(a) Make an appropriate construction in *GeoGebra* and study the position of point *M* for different positions of point *P*. Formulate a conjecture.

(b) Are you convinced of the truth of your conjecture? If not, try to use *GeoGebra* to support your conjecture. When you are convinced, go to the next task.

(c) Explain in your own words why your conjecture is true.

(d) Construct a proof.

(e) Investigate if your conjecture can be generalized. Perform the tasks above with new premises, by using appropriate techniques, such as posing *what if?* or *what if not?* questions.

Let *P* be an arbitrary point on an ellipse. Let *M* be the midpoint between P and one of the foci.

(a) Make an appropriate construction in *GeoGebra* and study the position of point *M* for different positions of point *P*.
Formulate a conjecture.

If *P* is an arbitrary point on an ellipse and *M* is the midpoint between *P* and one of the foci *F*, then the locus of *M* is an ellipse

(b) Are you convinced of the truth of your conjecture? If not, try to use *GeoGebra* to support your conjecture. When you are convinced, go to the next task.

(c) Explain in your own words why your conjecture is true.

(d) Construct a proof.

(e) Investigate if your conjecture can be generalized. Perform the tasks above with new premises, by using appropriate techniques, such as posing *what if?* or *what if not?* questions.

What if M is not the midpoint between P and F?

If *P* is an arbitrary point on an ellipse and *M an arbitrary point* on the line through *P* and one of the foci *F* (so that the ratio *FM/FP* is constant) then the locus of *M* is an ellipse

What if P is a point on another conic section?

If *P* is an arbitrary point *on a conic section* and *M* is an arbitrary point on the line through *P* and one of the foci *F* (so that the ratio *FM/FP* is constant) then the locus of *M* is *a conic section of the same kind*

What if M is a point between P and an arbitrary point (instead of F)?

If *P* is an arbitrary point on a conic section and *M* is an arbitrary point on the line through *P* and *an arbitrary point Q* (so that the ratio *QM/QP* is constant) then the locus of *M* is a conic section of the same kind

Is this about conic sections at all?
What if *P* is a point on another geometrical object?

If *P* is an arbitrary point on *an arbitrary geometrical object* and *M* is an arbitrary point on the line through *P* and an arbitrary point *Q* (so that the ratio *QM/QP* is constant) then the locus of *M* is similar to *this geometrical object*.

In mathematical theory this is a transformation termed homothety (a special case of similarity)

A model for task design with focus on exploration, explanation, and generalization in *GeoGebra*

Description of the mathematical situation

(a) Make an appropriate construction in *GeoGebra.* Formulate a conjecture.
(b) Are you convinced of the truth of your conjecture? If not, try to use *GeoGebra* to support your conjecture. When you are convinced, go to the next task.
(c) Explain in your own words why your conjecture is true.
(d) Construct a proof.
(e) Investigate if your conjecture can be generalized. Perform the tasks above with new premises, by using appropriate techniques, such as posing *what if?* or *what if not?* questions.

One further example

Let E be the extreme point of the function $f(x) = x^2 + bx$,
where b is a real number.

(a) Make an appropriate construction in *GeoGebra* and study the position of point E, for different values of the parameter b. Make a conjecture.

The subsequent tasks (b) - (e) could be the same as before.

Thank you for your attention!

Fahlgren, M. & Brunström, M. (2014). A model for task design with focus on exploration, explanation, and generalization in a dynamic geometry environment. *Technology, Knowledge and Learning*, *19*(3), 287–315.