Division for Engineering Sciences,
Physics and Mathemathics

3rd Workshop on Kinetic Theory and Applications
Karlstad University, Sweden, 15 – 17 June 2003

Abstracts

L.Caffarelli (Austin, TX): *Optimal transportation and some classical inequalities*
Abstract: We will discuss how the PDE properties of optimal transportation imply correlation and related inequalitites.

M. Benedicks (Stockholm): *Ergodic properties of Hénon maps and related dynamical systems*
Abstract: In this talk I will discuss the development in dynamical systems that started with the proof by Lennart Carleson and myself of chaotic behaviour for a family of Hénon maps. I in particular I will talk about the more recent development in the ergodic theory of these and related maps: existence of Sinai-Ruelle-Bowen measures, decay of correlation and stability under random perturbations (joint work with Lai-Sang Young and Marcelo Viana). I also plan to discuss some open problems that possibly can be addressed by these and related methods.

V.Kolyada (Madrid): *On Sobolev type inequalities*
Abstract: We study (anisotropic) Sobolev spaces of functions whose weak derivatives belong to L^p. For functions in these spaces, we consider sharp estimates of smoothness in L^q, ~$q>p$. In particular, we obtain inequalities between gradient norm and various Besov norms. We consider also a mixed Besov norms which have certain relation to averaging type lemmas often used in applications to kinetic equations and PDEs. We apply these results to prove some theorems on integrability of Fourier transforms.

K.Oskolkov (Columbia, SC): *Analytic number theory in Schroedinger type equations with the periodic initial data*
Abstract: The method of exponential sums is a recognized powerful tool of Analytic Number Theory. This method was created and developed in the classical works of Gauss, Riemann, Weyl, Hardy, Littlewood, Vinogradov, Lo Keng Hua and many other great mathematicians of the past, with the goal to find the solutions of the problems of number theory, such as Waring's and Goldbach's problems, the problem of distribution of primes (Riemann's conjecture), etc.
Recently, the method of exponential sums has found multiple applications in a quite different field - Mathematical Physics, namely, in the study of the properties of the solutions of the Cauchy initial value problem posed for a wide class of Schroedinger type equations. The simplest representatives is the Schroedinger equation of a free particle, and the degenerated (linearized) version of the Korteweg - deVries equation. If the initial data function is space periodic, then the solutions of such linear
equations exhibit the features that are commonly believed to be merely non-linear effects, such as turbulence. The main feature is self-similarity and non-trivial dimensions characteristics. The complete rational exponential sums (Gauss' sums of higher order) play the role of the scaling factors,

and the pattern is represented by the homogeneous functions, defined by the oscillatory integrals of the Fresnel or Airy type.
In the presentation, the following quantitative and qualitative results, concerning the properties of the solutions will be discussed. These results were established on the base of the application of the circular method of Vinogradov.

1) Hilbert transforms of exponents with the polynomial phase. Vinogradov's extensions.
2) Non-trivial global boundedness conditions. Relations with the incomplete Gauss' sums.
3) Local properties - continuity, differentiability.
4) Propagation of singularities.
5) Solutions of the equations with smooth potential.

Some more recent numerical results concerning the density distribution of the quantum particle with the elliptic theta-type initial function will be also presented. These results have been obtained in collaboration with Daniel Dix, they include 3d-graphing of the density function, and the graph of a family of the corresponding Bohmian trajectories.

P.Kurlberg (Göteborg): *Lattice points on circles and the discrete velocity model for the Boltzmann equation*
Abstract: In the context of the Discrete Velocity Model (DVM) for the Boltzmann equation in the plane, it is interesting to know whether lattice points on circles are angularly equidistributed. Using results from analytic number theory, namely certain bounds on mean values of multiplicative functions, we can show that lattice points on circles are angularly equidistributed {\em on average}, and from this it follows that the DVM is consistent. (Joint work with L. Fainsilber and B. Wennberg.)

F.Poupaud (Nice): *Effective equations in random media*
Abstract: We give in this talk a method to study the transport of particles in time-dependent random media. The method also allows to obtained homogenized limits of linear PDE's with random coefficients. The key point is a quasi-Markov assumption on these coefficients. This method has successfully been applied to show the convergence of a Liouville equation to a Fokker-Planck equation, to obtain the semi-classical limit of Schrödinger equations and the effective equations for passive scalars in turbulent flow.

A.Palczewski (Warsaw): *Stochastic particle system approximating Navier-Stokes like equation*
Abstract: We consider a Hamiltonian system of interacting particles. The potential of interaction is assumed to be a smooth function. In addition the system undergoes uncorrelated random perturbations. The character of perturbations enable formulation of equations describing time evolution of the system as a stochastic differential system of Ito type.
We reformulate this stochastic Hamiltonian system in terms of objective probability and look for equations governing the evolution of macroscopic quantities describing the system. Under reasonable assumptions of smallness of random perturbations we arrive to a system of partial differential equations which can be described as a Navier-Stokes type system with an additional term related to self-diffusion. This interesting asymptotic behaviour can be obtained only for a very restrictive class of interaction potentials.

L.Arkeryd (Göteborg): *A stationary kinetic two-rolls system of Boltzmann type and related problems*
Abstract: The presentation will discuss joint work with A. Nouri on perturbative two-rollsproblems and corresponding small mean free path limits. The focus is on a particular situation with bifurcation and multiple positive solutions, including leading order hydrodynamic limits, in a perspective of more general set-ups and the resolution of a variety of asymptotic stationary questions.

G.Toscani (Pavia): *Asymptotic properties of the inelastic Kac model*

Abstract: We discuss the asymptotic behavior of the solution to certain models of dissipative systems obtained from a suitable modification of Kac caricature of a Maxwellian gas. It is shown that global equilibria different from concentration are possible if the energy is not finite. These equilibria are distributed like stable laws, and attract initial densities which belong to the normal domain of attraction. If the initial density is assumed of finite energy, with higher moments bounded, it is shown that the solution converges for large--time to a profile with power law tails. These tails are heavily dependent of the collision rule. Our results extend the analysis given for pseudo-maxwellian inelastic gas in [1], and give a partial answer to the question posed by Ernst and Brito in [2]. (Joint work with Ada Pulvirenti).

[1] A.V. Bobylev, C. Cercignani, G.Toscani, Proof of an asymptotic property of self-similar solutions of the Boltzmann equation for granular materials. J. Statist. Phys., 111 (2003) 403-417.

[2] M.H. Ernst, R. Brito, Scaling solutions of inelastic Boltzmann equation with over-populated high energy tails. J. Statist. Phys., 109 (2002) 407-432.

G.Spiga (Parma): *Kinetic theory with non-conservative interactions*

G.Russo (Catania): *Variable grid methods for kinetic equations*

Abstract: This talk presents some preliminary results on deterministic numerical methods for approximation of kinetic equations. The goal is to obtain numerical schemes with a grid in velocity space which scales with the solution, and which is therefore different at different space locations. The technique is based on the use of scaled velocity variable. The distribution function, expressed in terms of the scaled velocity, satisfies a Boltzmann-like equation, with an additional drift term. Preliminary results on space-homogeneous Boltzmann equation for granular flow, and on one dimensional BGK model will be presented.

K.Aoki (Kyoto): *Fluid-dynamic system for a mixture of a vapor and a noncondensable gas*

Abstract: In [TA], by means of a formal but systematic asymptotic analysis based on the Boltzmann equation, we derived a system of fluid-dynamic equations and boundary conditions describing the steady behavior of a mixture of a vapor and a noncondensable gas around arbitrarily shaped boundary, consisting of the condensed phase of the vapor and thus allowing its evaporation and condensation, at small Knudsen numbers (near continuum regime). To be more precise, the system covers the situation where the temperature and density variations are large, but the Mach number of the flow is as small as the Knudsen number. This fluid-dynamic system is an extension of the system derived for a single-component gas in [SATSB]. Therefore, the system contains the ghost effect. That is, in spite of the fact that the flow vanishes in the continuum limit (the limit where the Knudsen number vanishes), its trace still has a finite effect on the density and temperature fields in the same limit.

The system derived in [TA] was, however, still far from practical applications for the following reasons: (i) The fluid-dynamic equations contain various transport coefficients, which are unknown functions of the local concentration of the vapor (i.e., the local number density of the vapor divided by that of the total mixture); (ii) The boundary conditions contain two slip coefficients (the coefficient of thermal creep and that of diffusion slip), which are also unknown functions of the local concentration of the vapor.

In recent studies, we have resolved these problems and made the fluid-dynamic system applicable to practical computations. More precisely, we have built a database of the transport coefficients in [TYAS] and a database of the slip coefficients in [TYKA]. In the latter paper, the problems of thermal creep and diffusion slip for a binary mixture, which are fundamental half-space boundary-value problems for the linearized Boltzmann equation for a gas mixture, are solved accurately by means of a finite-difference method. Incidentally, a related mathematical study of the half-space problems (Milne and Kramers problems) for the linearized Boltzmann equation for a mixture is performed in [ABT].

In this talk, we will give a brief overview of this fluid-dynamic system.

References:
SATSB : Y. Sone, K. Aoki, S. Takata, H. Sugimoto, and A. V. Bobylev, Phys. Fluids 8, 628 (1996).
TA : S. Takata and K. Aoki, Transp. Theor. Stat. Phys. 30, 205 (2001).
TYAS : S. Takata, S. Yasuda, K. Aoki, and T. Shibata, in Rarefied Gas Dynamics, edited by E. P. Muntz and A. Ketsdever (AIP, Melville, 2003) (to be published).
TYKA : S. Takata, S. Yasuda, S. Kosuge, and K. Aoki, Phys. Fluids (submitted).
ABT : K. Aoki, C. Bardos, and S. Takata, J. Stat. Phys. 112 (2003) (to be published).

L.Söderholm (Stockholm): *The transverse (Magnus force) on a rotating sphere in a rarefied gas*

Abstract: Writing the abstract in the day when the Swedish golf player Annika Sörenstam meets the male elite in golf seems appropriate.
The spin of a golf ball will cause the ball to take on a curved orbit. If you see the ball approaching you it will deviate in the same direction as you see it rotate. This so called Magnus effect is a well studied phenomen within the realm of fluid dynamics. It has an important role in ball games like golf an soccer. In this work the corresponding phenomen, the motion of a rotating sphere in a rarefied gas is considered is considered. We study the limit where the radius of the sphere is small compared to the mean free path of the gas. We conclude that in a sufficiently rarefied gas a ball will deviate in the direction opposite to the direction you see it rotate. The Magnus effect is reversed. The influence o this reversed Magnus effect on the orbit of a rapidly spinning satellite is also calculated. (Joint work with Karl Borg and Hanno Essén)

A.Heintz (Göteborg): *Willmore geometric flows. A kinetic approach*

Abstract: Geometric flows of surfaces tending to minimize energies depending of the curvature are considered. Typical example of such an energy is Willmore functional: integral of the square of the mean curvature, that represents bending energy of a thin shell. Corresponding Euler-Lagrange operator is a complicated nonlinear operator of the fourth order of coordinates. Minima of the Willmore functional - so called Willmore surfaces were a popular subject in geometry during last 20 years. Willmore geometric flow is a gradient flow corresponding to Willmore functional. We suggest a simple kinetic approximate scheme to a family of geometric flows including Willmore flows interesting for applications in biofisics and discuss corresponding mathematical problems and numerical results. (Joint work with Richard Grzibovski)

R.Pettersson (Göteborg): *On solutions to the linear Boltzmann equation for granular gases*

Abstract: We consider the time- and space-dependent linear Boltzmann equation with general boundary conditions in the case of inelastic (granular) collisions. First mild L1-solutions are constructed as limits of iterate functions. Then boundedness of all higher velocity moments are obtained. Finally the question of convergence to equilibrium is studied, using a general H-theorem for a relative entropy functional.

A.Nouri (Marseille): *On a quantum kinetic equation linked to the Compton effect*

Abstract: Quantum effects describing the interaction between photons and electrons can be studied by an homogeneous kinetic equation. Depending on the initial data, cases of long-time existence of solutions with asymptotic convergence to equilibrium and of non-existence of solutions are proven.

S.Calogero (Potsdam): *The Nordstrom-Vlasov system*

Abstract: The Nordstrom-Vlasov system describes the dynamics of a self-gravitating ensemble of collisionless particles in the framework of a relativistic scalar theory of gravitation. Although this is not a physically correct model, it is interesting because it still captures some of the essential features which distinguish the non-relativistic model, the Vlasov-Poisson system, from the correct relativistic model, the Einstein-Vlasov system. Examples of such features are the non-linearity of the field equation and the propagation of gravitational waves due to the hyperbolic character of the system. In this talk I will give a short introduction to the Nordstrom-Vlasov system and present some results concerning the existence of classical solutions and of spherically symmetric equilibria with finite radius.

B.Wennberg (Göteborg): *Deriving the Boltzmann equation from a lattice gas*

Abstract: G. Gallavotti made a rigorous derivation of a linear Boltzmann equation from the Lorentz gas. The Lorentz gas describes the dynamics of a single particle moving in straight lines between circular scatterers on which the particle is specularly re ected. In the paper of Gallavotti, scatters of radius r are distributed according to a Possion process in the plane with density 1/r, and the Boltzmann equation is obtained in the limit when
$r \to 0$. In a purely periodic distribution of scatters, it is not possible to derive a Boltzmann equation, but Caglioti, Pulvirenti and Ricci analysed a random, but periodic distribution, and proved that in that case, the limit is a Boltzmann equation. Motivated by this, we have studied a family of models intermediary between the case studied by Caglioti et al, and the purely periodic distribution. This model was presented at the meeting in Karlstad 2002. At that time, we had proved that a "Markovian modification" of the Lorentz gas does indeed converge as expected. To prove the desired result, we also needed a result stating that the Markovian version is in the limit equivalent to the Lorentz gas. In the talk, I will brie y review the ideas behind the Lorentz gas and the limiting procedure leading to the Boltzmann equation. And I will in discuss the difference between the Lorentz gas and the Markovian version, and show that they are equivalent in the limit. (Joint work with Valeria Ricci).