Division for Engineering Sciences,
Physics and Mathemathics

4th Workshop on Kinetic Theory and Applications
Karlstad University, Sweden, 12 – 14 June 2005

Abstracts

Y. Brenier (Paris): *Optimal transportation: theory and applications*
Abstract: The first optimal transportation problem was addressed by Monge in 1781 in his ``memoire sur la theorie des deblais et des remblais'', a Civil Engineering problem where parcels of materials have to be displaced from one site to another one with minimal transportation cost. A modern treatment of this problem was initiated by Kantorovich in 1942, based on a probabilistic approach to reduce it to an infinite dimensional linear program. In the late 80', a new connection was made between Optimal Transportation Theory and non-linear Partial Differential Equations. In particular, a variant of the original Monge transportation problem was related to two of the most interesting and challenging (both analytically and numerically) non-linear PDEs, namely the (real) Monge-Amp`ere equations and the Euler equation of incompressible inviscid fluids. Rapidly, optimal transportation has turned out to be a powerful concept, giving new insights on both Analysis (functional inequalities, Log-Sobolev inequalities, geometry of convex bodies...) and Modelling (inverse problems in computational Cosmology and Geophysics, image processing...). The aim of the talk is to give some flavour of this flourishing field of Pure and Applied Mathematics.

K. Oskolkov (Columbia, SC): *On a problem of Chowla and the Schrödinger density*

V. Kolyada (Karlstad): *Mixed norms, multiplicative inequalities and embedding theorems*

B. Wennberg (Göteborg): *Convergence to a stationary state for the Kac equation with a force field*
Abstract: So called Gaussian thermostats have been introduced for example in molecular dynamics as a means of studying non-equilibrium steady states. I will present some results concering the Kac equation, and in particular a proof that the solutions converge to a stationary, non-equilibrium state."

H. Andreasson (Göteborg): *On gauge conditions and continuation criteria for the spherically symmetric Einstein-Vlasov system*
Abstract: The form of the Einstein equations depend on the choice of gauge and we will briefly discuss this in the general and in the spherically symmetric case. An improved continuation criterion for global existence in Schwarzschild coordinates (polar time slicing gauge and areal radial coordinate) will then be presented.

M. Pulvirenti (Rome): *To be announced*

R. Esposito (Rome): *On the derivation of the Boltzmann equation from the Schrödinger equation: partial results*

I. Gamba (Austin, TX): *Point-wise bounds for solutions of homogeneous Boltzmann equations*

Abstract: (in collaboration with Vladislav Panferov and Cedric Villani) We consider space homogeneous Boltzmann collisional models for, both, elastic (energy conservative) or inelastic (energy dissipative) particle interactions. We have recently derived, in collaboration with Bobylev and Panferov, a sharper Povzner type estimates which, essentially, provides weighted-L^1 control of solutions to kinetic equations with classical bilinear collisional integral forms for variable hard potentials with integrable symmetrical, radially decreasing angular cross sections (This part was an extension of the work of Bobylev'97 for elastic hard spheres models). This estimate, combined with a maximum principle and analysis of the Carlemann representation of the gain operator, provides the tool to obtain point-wise bounds solutions of these type of Boltzmann equations. These bounds are controlled by integrable functions with Gaussian exponential decay in the case of elastic energy conservative interactions, but they must be slower than classical Gaussians distributions for the case of inelastic or energy dissipative interactions. The decay exponent depends only on the balance between the forced term and the loss operator corresponding to the problem under consideration.

G. Toscani (Pavia): *Self-similarity and power-like tails in nonconservative kinetic models*

Abstract: We discuss the large-time behavior of solution of a simple kinetic model of Boltzmann-Maxwell type, such that the temperature is time decreasing and/or time increasing. We show that, under the combined effects of the nonlinearity and of the time-monotonicity of the temperature, the kinetic model has non trivial quasi-stationary states with power law tails. In order to do this we consider a suitable asymptotic limit of the model yielding a Fokker-Planck equation for the distribution. The same idea is applied to investigate the large-time behavior of a elementary kinetic model of economy involving both exchanges between agents and increasing and/or decreasing of the mean wealth. In this last case, the large--time behavior of the solution shows a Pareto power law tail. Numerical results confirm the previous analysis.

R. Monaco (Turin): *BGK-type models for reactive gas mixtures*

Abstract: Chemically reactive gas mixtures play a relevant role in many meaningful physical applications, as for instance combustion, plasma physics and air pollution, for which kinetic models of the extended Boltzmann equation can be used to improve the knowledge of the involved macroscopic phenomena, starting from a description of the system at the mesoscopic scale.
The generalisation of the Boltzmann equation to reactive gas mixtures is widely investigated in literature. Due to the mathematical complexity of the collision operators a large piece of research works has been addressed to simplified kinetic models, as for instance BGK-type models. Extensions of BGK-type models to multi-component systems can be found in [1], and more recently in [2]. These two papers are concerned with mixtures of inert gases.
Therefore, it seems to be a new interesting topic to deal with a BGK approximation of the extended Boltzmann equation for chemically reacting gases. In the context of strong reaction regime, two recent BGK-type approaches are available for different choices of Maxwellian distributions. In [3], a first euristic approach was based on Maxwellians depending on mass and number densities of each gas species, common velocity and temperature. Conversely, in [4], another approach was performed involving Maxwellians dependent on masses, number densities, velocity and temperature of each gas species.
In the present paper two BGK-type models, compatible with a slow and a fast chemical reaction, are derived. Such models depend on local Maxwellian distributions defined by masses and number densities of each species, global mean velocity and temperature of the mixture.
It is proven that both models verify indifferentiability principle, conservation of mass, momentum and total energy (kinetic plus chemical bond energy). The entropy inequality is assured under a suitable condition for the first model, and without any condition for the second one.

The behavior of the models is tested with respect to their trend to equilibrium. Numerical experiments are given for both fast reactions, typical of combustion processes, and for slow ones occurring, in general, in enviromental pollution.

[1] V. Garzo', A. Santos, J.J. Brey, A kinetic model for a multicomponent gas, *Phys. Fluids*, 1, p.380, 1989.
[2] P. Andries, K. Aoki, P. Perthame, A consistent BGK-type model for gas mixtures, *J. Stat. Phys.*, 106, p.993, 2002.
[3] R. Monaco, M. Pandolfi Bianchi, A BGK-type model for a gas mixture with reversible reactions, in *New Trends in Mathematical Physics*, Eds. P. Fergola et al., World Scientific, Singapore, 107, 2004.
[4] M. Groppi, G. Spiga, A BGK-type type approach for chemically reacting gas mixture, *Phys. Fluids*, 16, p.4273, 2004.

S. Rjasanow (Saarbrücken): *Time splitting schemes for the inelastic Boltzmann equation*
Abstract: In the present paper we consider three different stochastic numerical algorithms for the spatially homogeneous uniformly heated granular Boltzmann equation. The first scheme is a generalization of the Bird's DSMC method [1] to the case of inelastic collisions with additional Browning motion. This scheme does not contain time discretisation parameter, it is exact. This scheme was introduced in [2]. The second scheme is a simple splitting procedure having the formal accuracy of the order Δt. The third scheme is the Strang splitting scheme of the second order with respect to Δt. The accuracy of these three algorithms will be studied by the use of the analytical relaxation of the temperature in the case of Maxwell molecules.

1 : Bird G. A., *Molecular Gas Dynamics and the Direct Simulation of Gas Flows*., Clarendon Press, Oxford, 1994.
2 : Rjasanow S., Gamba I. M. and W. Wagner. Direct simulation of the uniformly heated granular Boltzmann equation., *Math. Comput. Modelling*, 2005.

K. Aoki (Kyoto): *Cylindrical Couette flow of a vapor-gas mixture: A ghost effect in the fluid-dynamic limit*
Abstract: (in collaboration with H. Yoshida) A binary mixture of gases between two coaxial rotating circular cylinders is considered. One of the component gases is a vapor of the substance forming the cylinders, so that it may evaporate or condense (or sublimate) on the cylinders. Instead, the other component is noncondensable in the sense that it neither evaporates nor condenses on the cylinders. The steady behavior of the mixture is investigated on the basis of kinetic theory with special interest in the case of small Knudsen numbers (i.e., near continuum regime). First, a formal but systematic asymptotic analysis of the Boltzmann system is carried out, and a system of fluid-dynamic equations and their boundary conditions describing the behavior of the mixture in the continuum limit is derived. The system shows the following properties. (i) Evaporation and condensation of the vapor stop in the limit; however, the evaporation and condensation of infinitesimal strength have a finite effect on the other quantities, such as the circumferential component of the flow velocity, in the limit (the ghost effect); (ii) a nonuniqueness (or a bifurcation) of the solution arises. The property (ii) is a consequence of numerical analysis of the fluid-dynamic system. Second, a Monte-Carlo simulation of the original Boltzmann system is performed for small Knudsen numbers, and the result that is consistent with the properties (i) and (ii) is obtained.

Hybrid Burnett Equations. A New Method of Stabilizing

Lars H. Söderholm
Mekanik, KTH, SE-100 44 Stockholm, Sweden
lars.soderholm@mech.kth.se
(Dated: April 15, 2005)

In the original work by Burnett the pressure tensor and the heat current contain two time derivates. Those are commonly replaced by spatial derivatives using the equations to zero order in the Knudsen number. The resulting conventional Burnett equations were shown by Bobylev to be linearly unstable. In this paper it is shown that the original equations of Burnett have a singularity. A hybrid of the original and conventional equations is constructed which is shown to be linearly stable. It contains two parameters. For the simplest choice of parameters the hybrid equations have no third derivative of the temperature but the inertia term contains second spatial derivatives. For stationary flow, when terms Kn^2Ma^2 can be neglected, the only difference from the conventional Burnett equations is the change of coefficients $\varpi_2 \to \varpi_3, \varpi_3 \to \varpi_3$.

M. Azadzadeh (Göteborg): *Chebyshev spectral-S_N methods for the neutron transport*
Abstract: (in collaboration with A. Kadem)

A. Heintz (Göteborg): *Geometric flows with boundary conditions. A convolution-thresholding approach*
Abstract: We consider a convolution-thresholding approach to boundary value problems for generalized curvature flows. The evolving surface is supposed to move inside a smooth domain in such a way the surface has a prescribed angle with the normal to the boundary of the domain at the intersection points. A motivation of the method from the point of view of kinetic equations is presented.

G. Spiga (Parma): *Moment equations and hydrodynamics for dilute granular flows*
Abstract: In the last few years, kinetic models for granular materials have attracted significant interest in the mathematical and physical communities. The largest attention has been devoted to nonlinear models for dilute flows based on generalizations of the Boltzmann-Enskog equation describing energy dissipation in binary encounters. Most of the analytical investigation has been performed in the frame of the so-called pseudo-Maxwellian inelastic particles, both in the free case in which granular temperature decays to zero, and in the driven case in which there is an external energy supply. Some results are available also for the more realistic, but much more difficult to deal with, inelastic hard sphere model. Very important for practical applications are also linear kinetic equations for dissipative interactions of dilute granular matter with a given background, though this kind of inelastic process has been much less addressed. The relevant physical problem describes typically the diffusion of fine polluting powders in a medium (like air). Due to momentum and energy exchange with the host medium, a collision equilibrium is possible, and it is reasonable to expect stability and relaxation to it (H-theorem). Equilibrium temperature has been shown to be intermediate between zero and the background temperature. First results for the pseudo-Maxwellian model have been extended to the hard sphere model, and significant existence results have been proved. We shall proceed along this research line by consistently deriving and investigating a macroscopic description in terms of suitable expansions, which allow closure of the moment equations for the most relevant physical fields. Moreover, we plan to achieve, in the asymptotic limit when an appropriate Knudsen number tends to zero, hydrodynamic equations at the Navier-Stokes level for the conserved quantities.

A. Nouri (Marseille): *On Bose-Einstein condensation*
Abstract: A system coupling the condensate density to the non-condensate distribution function of a gas at very low temperature is considered, in a space-homogeneous setting. A global existence in time of a solution to the Cauchy problem is proven for an initial datum with finite mass and energy. Other models are described.